Davide Osenda

Last lesson in Göttingen

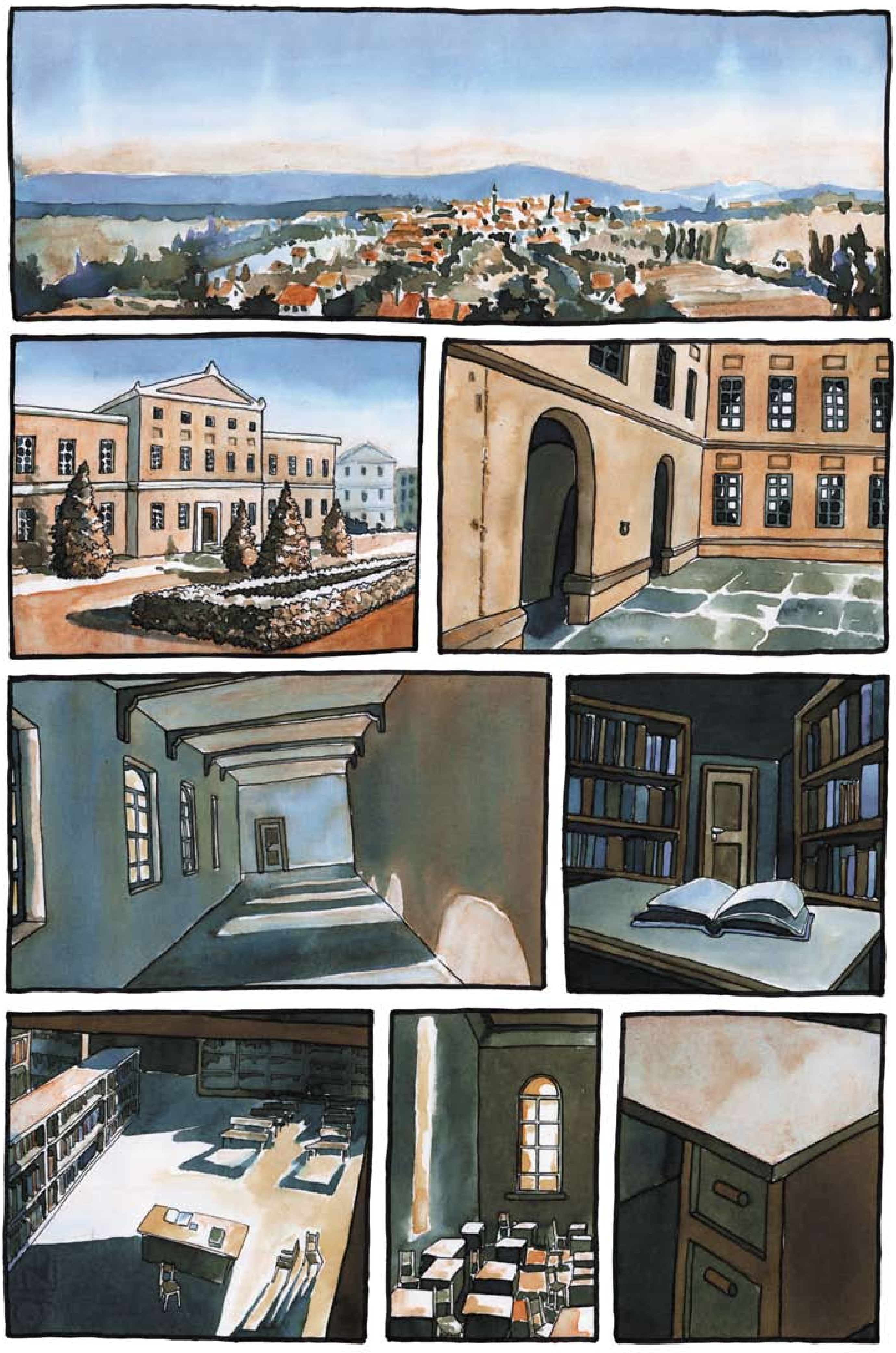
Written and illustrated by Davide Osenda.
Translated by Luisa Pellegrino and Ilaria Oddenino.

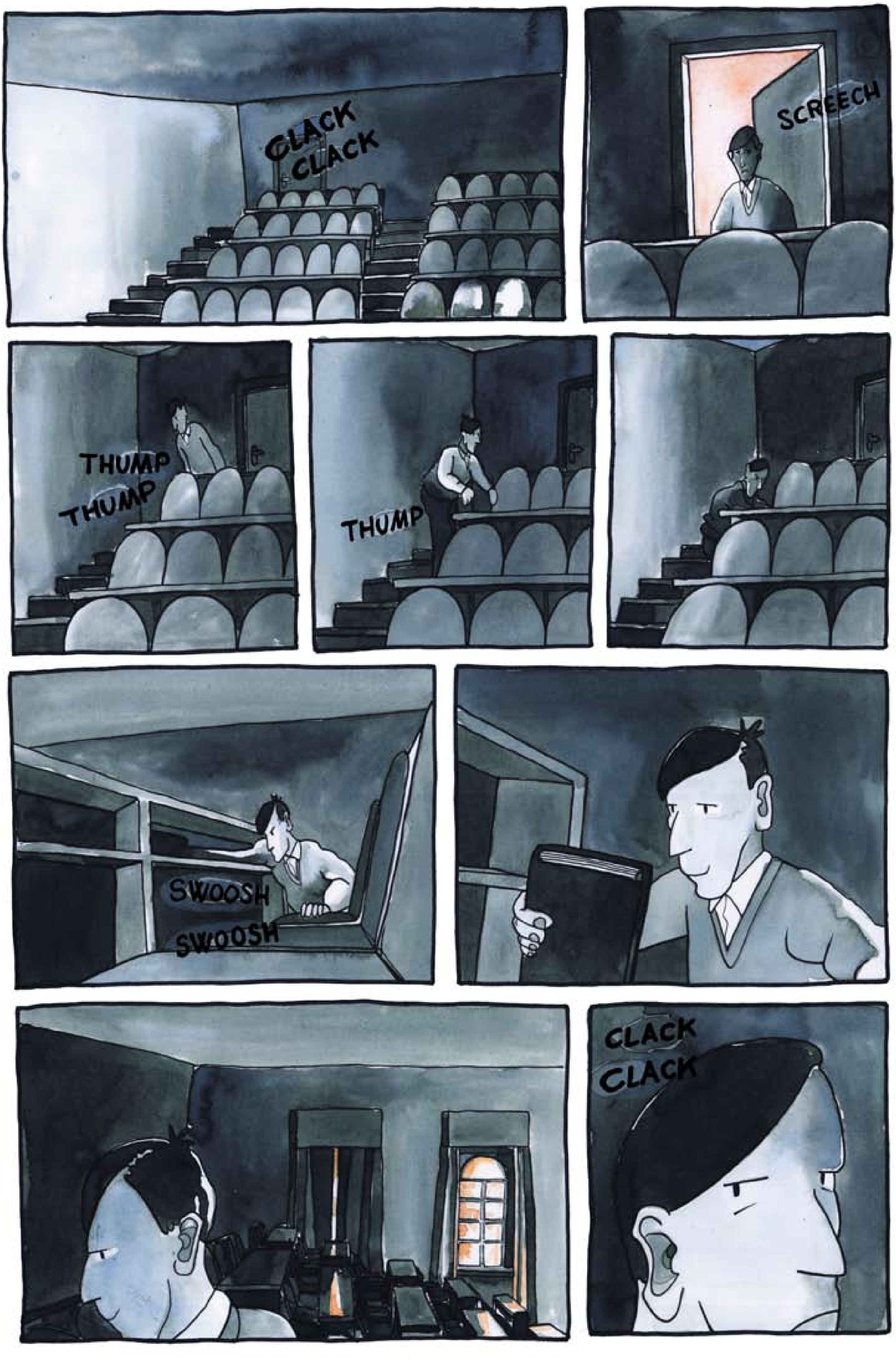
Luisa Pellegrino lives in Turin. She is a cook at Pilou (www.pilou.it). Sometimes she translates and writes about literature. In 2012 she received a PhD in English and Postcolonial Literature from the University of Turin.

Ilaria Oddenino is a teacher and literary translator. In 2011 she received her PhD in English and Postcolonial Literature from the University of Turin, where she currently lives.

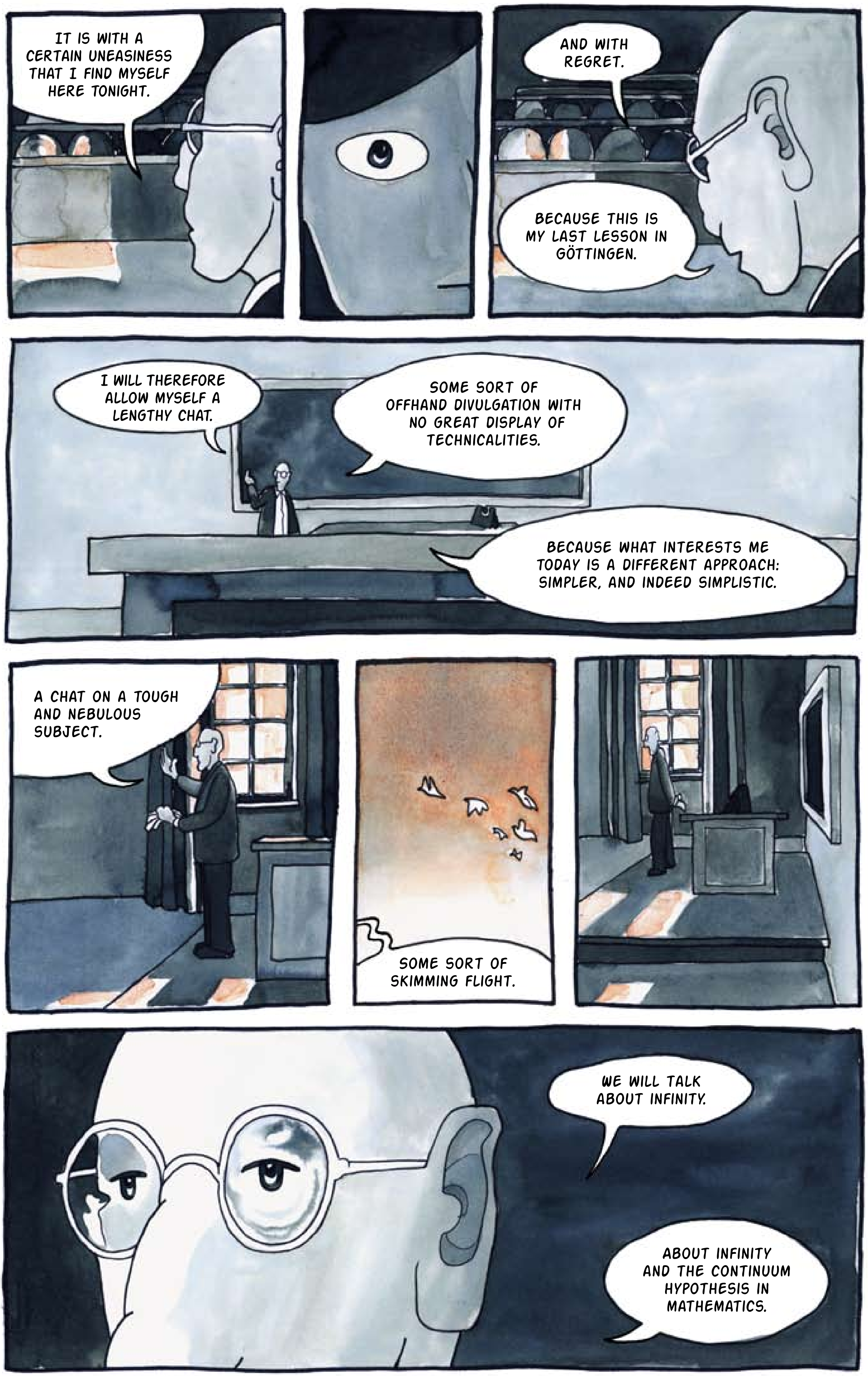
Last lesson in Göttingen

Chapter 1

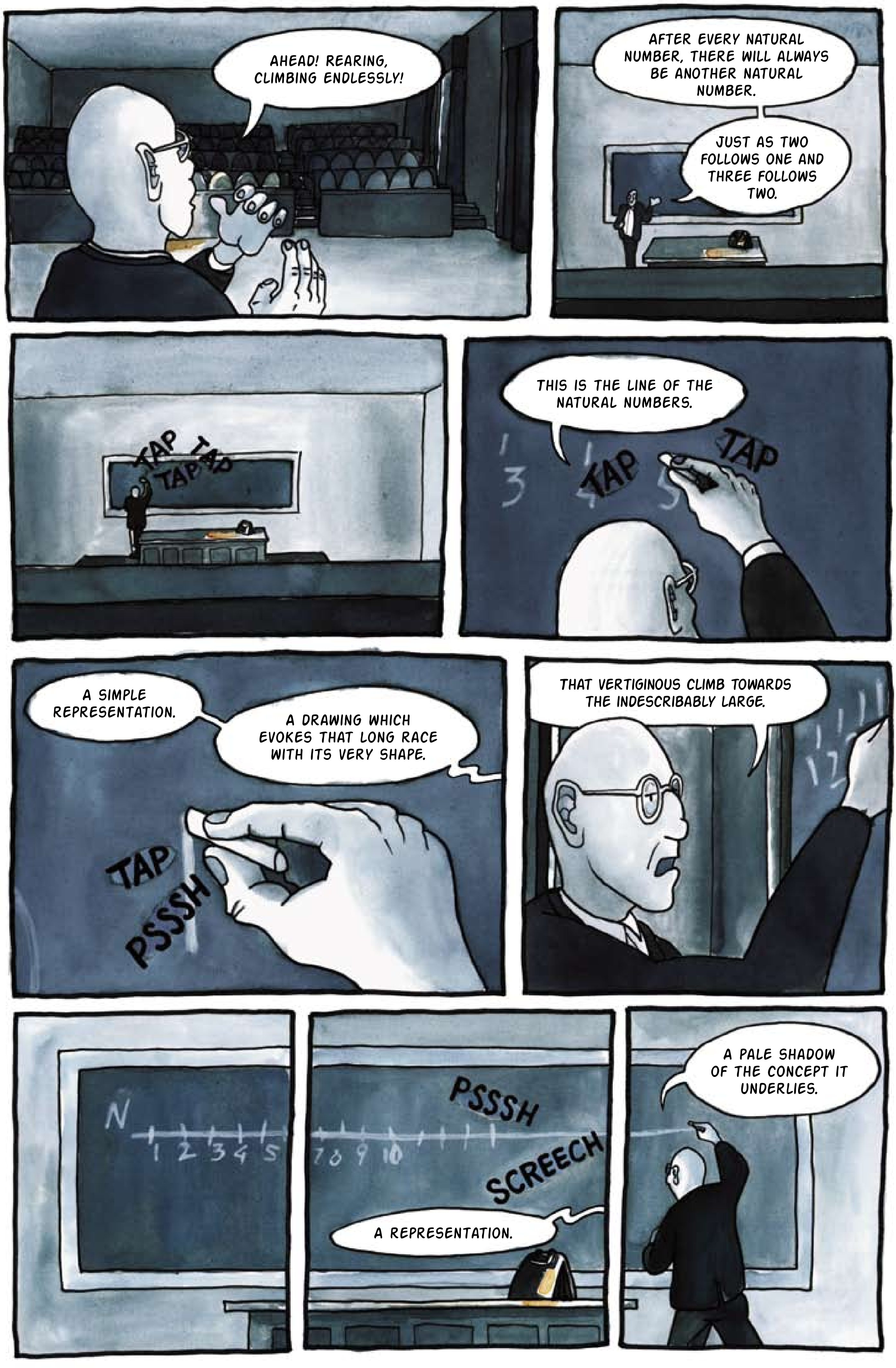


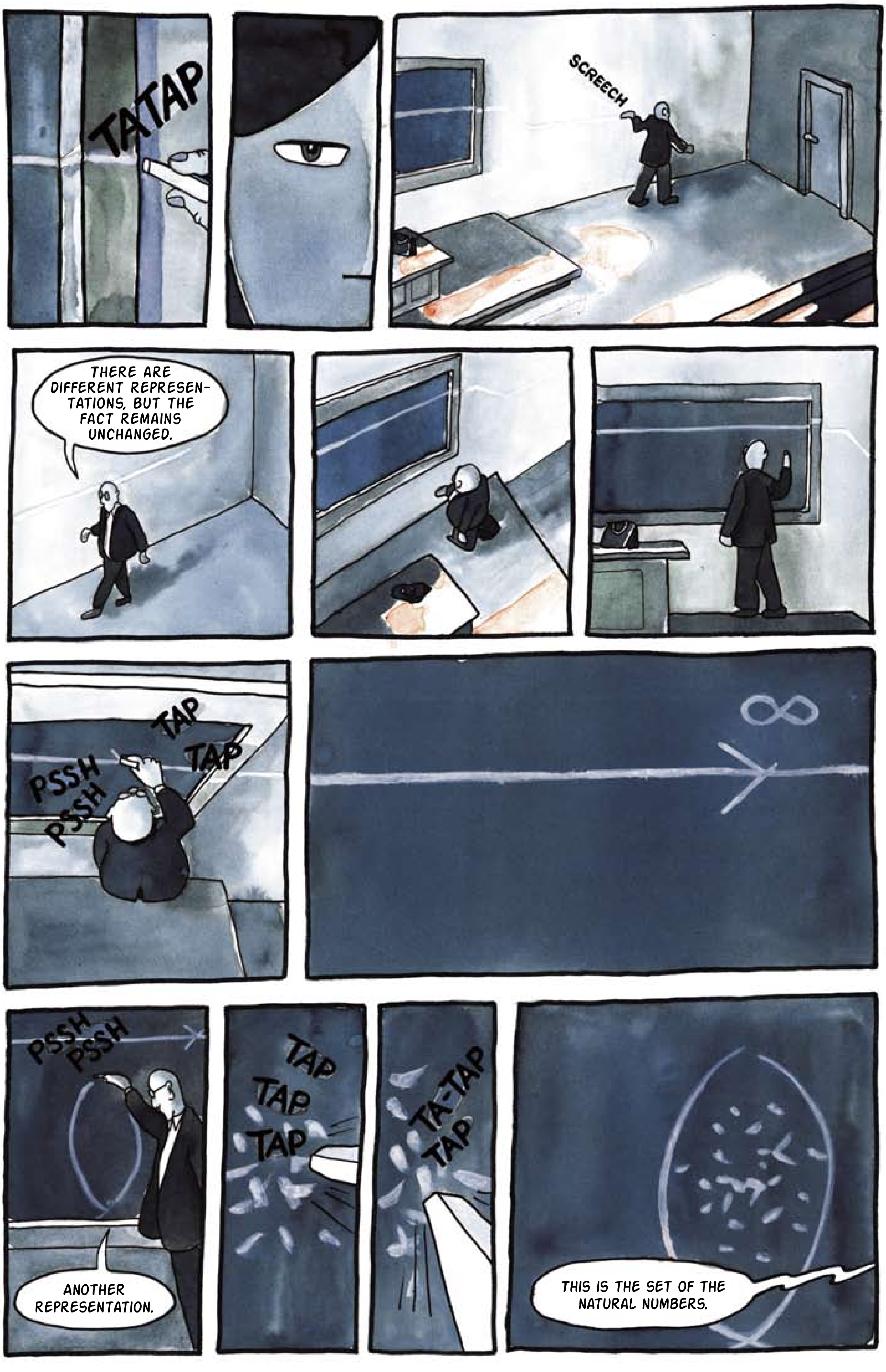

CLACK
CLACK
SCREECH
THUMP
THUMP
THUMP
SWOOSH
SWOOSH
CLACK
CLACK

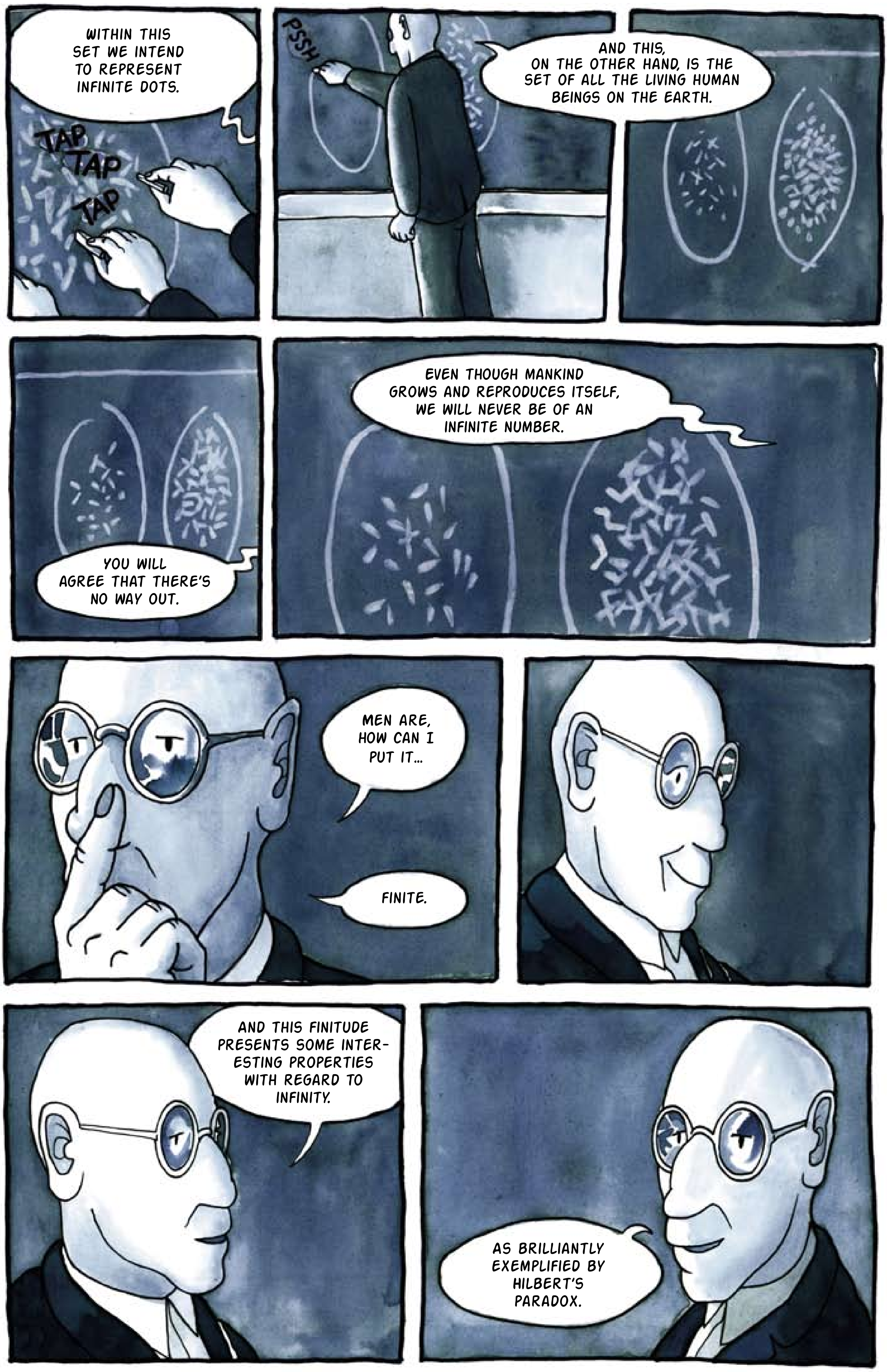

SCREECH
TAP
TAP
SWOOSH
SWOOSH
SWOOSH

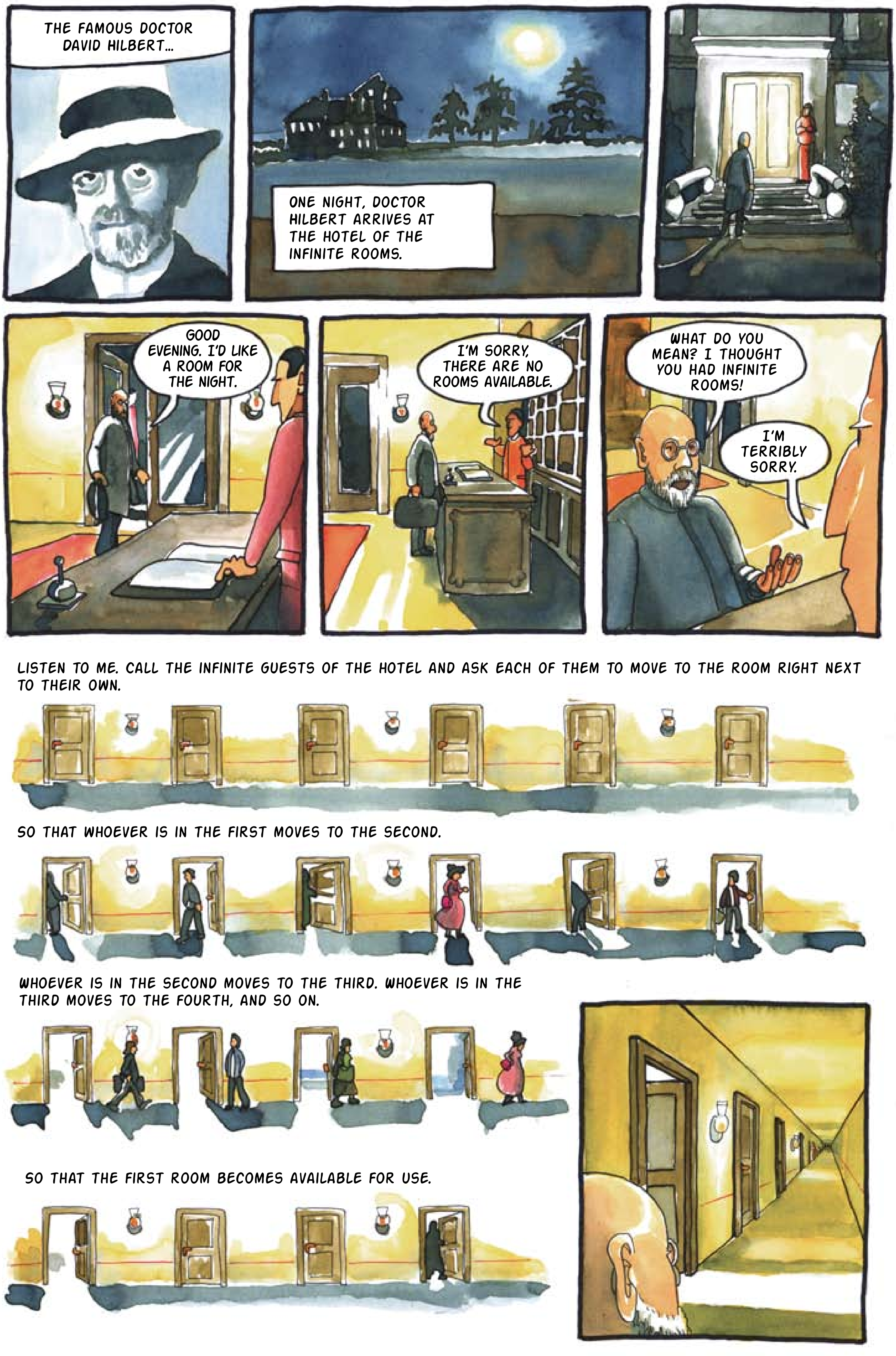

IT IS WITH A CERTAIN UNEASINESS THAT I FIND MYSELF HERE TONIGHT.
AND WITH REGRET.
BECAUSE THIS IS MY LAST LESSON IN GÖTTINGEN.
I WILL THEREFORE ALLOW MYSELF A LENGTHY CHAT.
SOME SORT OF OFFHAND DIVULGATION WITH NO GREAT DISPLAY OF TECHNICALITIES.
BECAUSE WHAT INTERESTS ME TODAY IS A DIFFERENT APPROACH: SIMPLER, AND INDEED SIMPLISTIC.
A CHAT ON A TOUGH AND NEBULOUS SUBJECT.
SOME SORT OF SKIMMING FLIGHT.
WE WILL TALK ABOUT INFINITY.
ABOUT INFINITY AND THE CONTINUUM HYPOTHESIS IN MATHEMATICS.

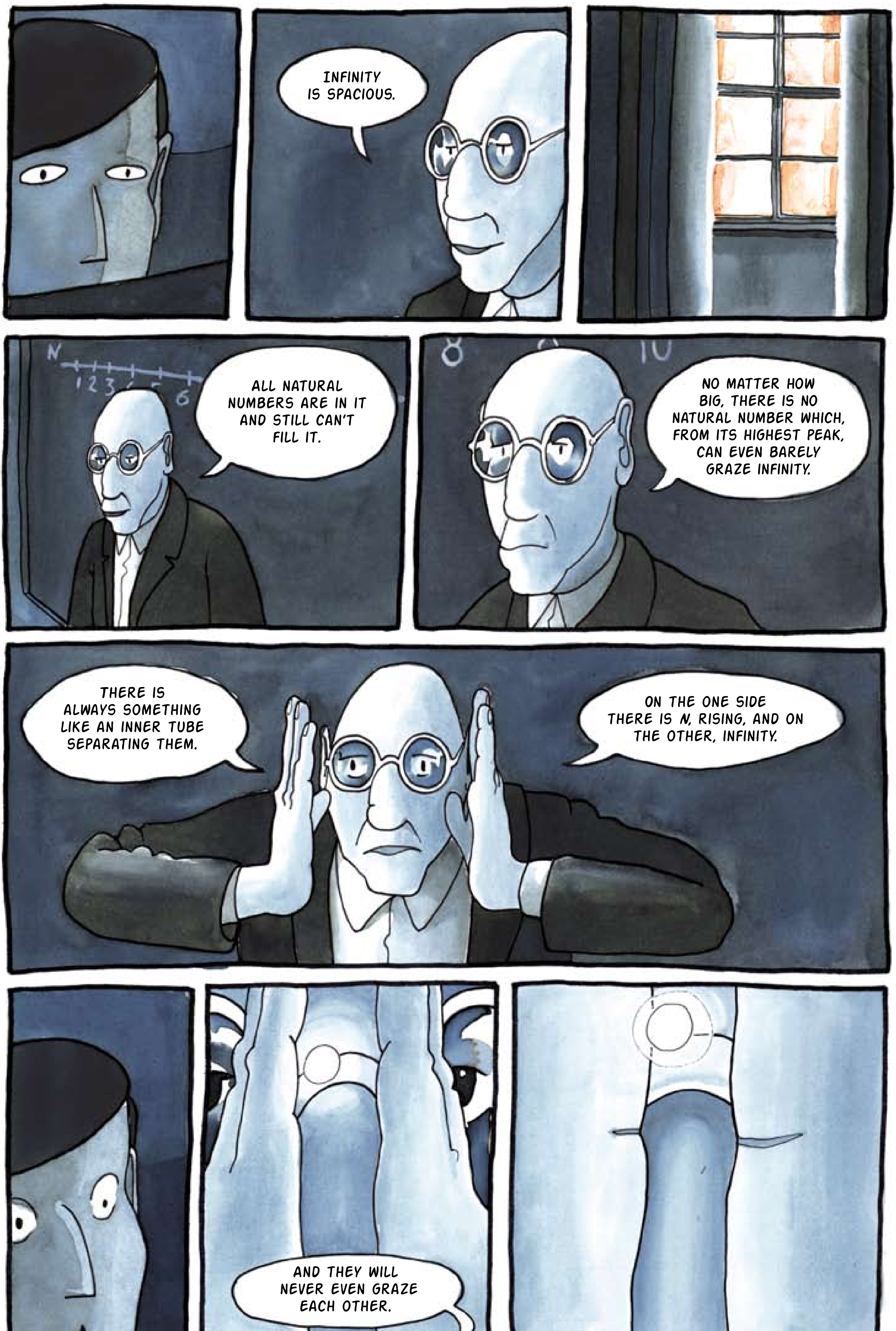

LET'S START WITH NUMBERS.
WITH THE EXPERIENCE WE HAVE OF NUMBERS.
THE NATURAL NUMBERS WHICH, PROGRESSIVELY, FOLLOW ONE ANOTHER.
AND GROW.
BOTH IN VALUE AND IN QUANTITY.
ONE.
TWO.
THREE.
FOUR.
FIVE.
TEN. TWENTY.
AND A HUNDRED. AND A THOUSAND. AND SO ON!


AHEAD! REARING, CLIMBING ENDLESSLY!
AFTER EVERY NATURAL NUMBER, THERE WILL ALWAYS BE ANOTHER NATURAL NUMBER.
JUST AS TWO FOLLOWS ONE AND THREE FOLLOWS TWO.
TAP TAP TAP
THIS IS THE LINE OF THE NATURAL NUMBERS.
TAP TAP
A SIMPLE REPRESENTATION.
A DRAWING WHICH EVOKES THAT LONG RACE WITH ITS VERY SHAPE.
TAP
PSSSH
THAT VERTIGINOUS CLIMB TOWARDS THE INDESCRIBABLY LARGE.
N
1 2 3 4 5 7 8 9 10
PSSSH
SCREECH
A REPRESENTATION.
A PALE SHADOW OF THE CONCEPT IT UNDERLIES.

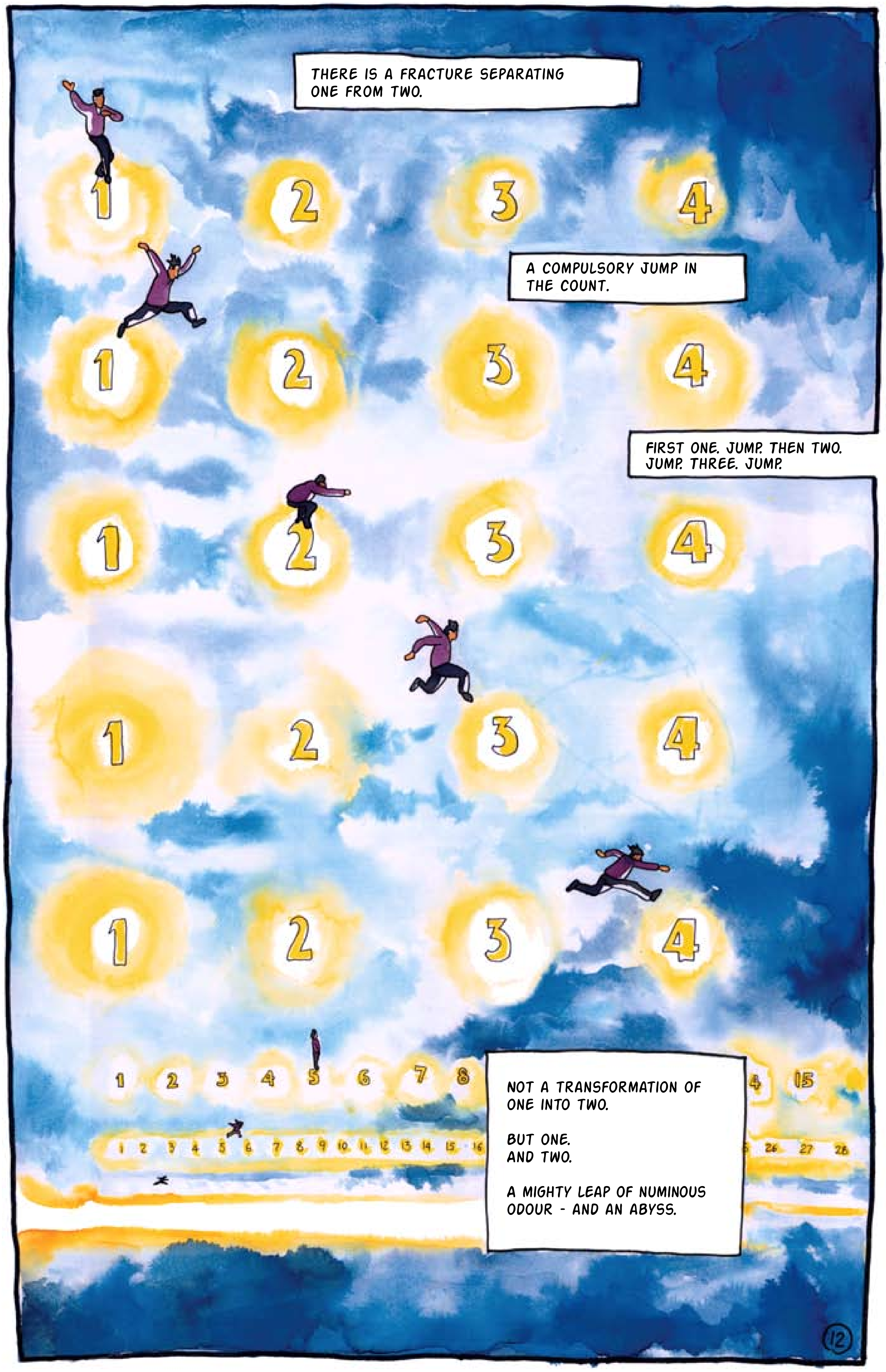

TATAP
SCREECH
THERE ARE DIFFERENT REPRESENTATIONS, BUT THE FACT REMAINS UNCHANGED.
TAP
TAP
PSSH
PSSH
PSSH
PSSH
ANOTHER REPRESENTATION.
TAP
TAP
TAP
TA-TAP
TAP
THIS IS THE SET OF THE NATURAL NUMBERS.

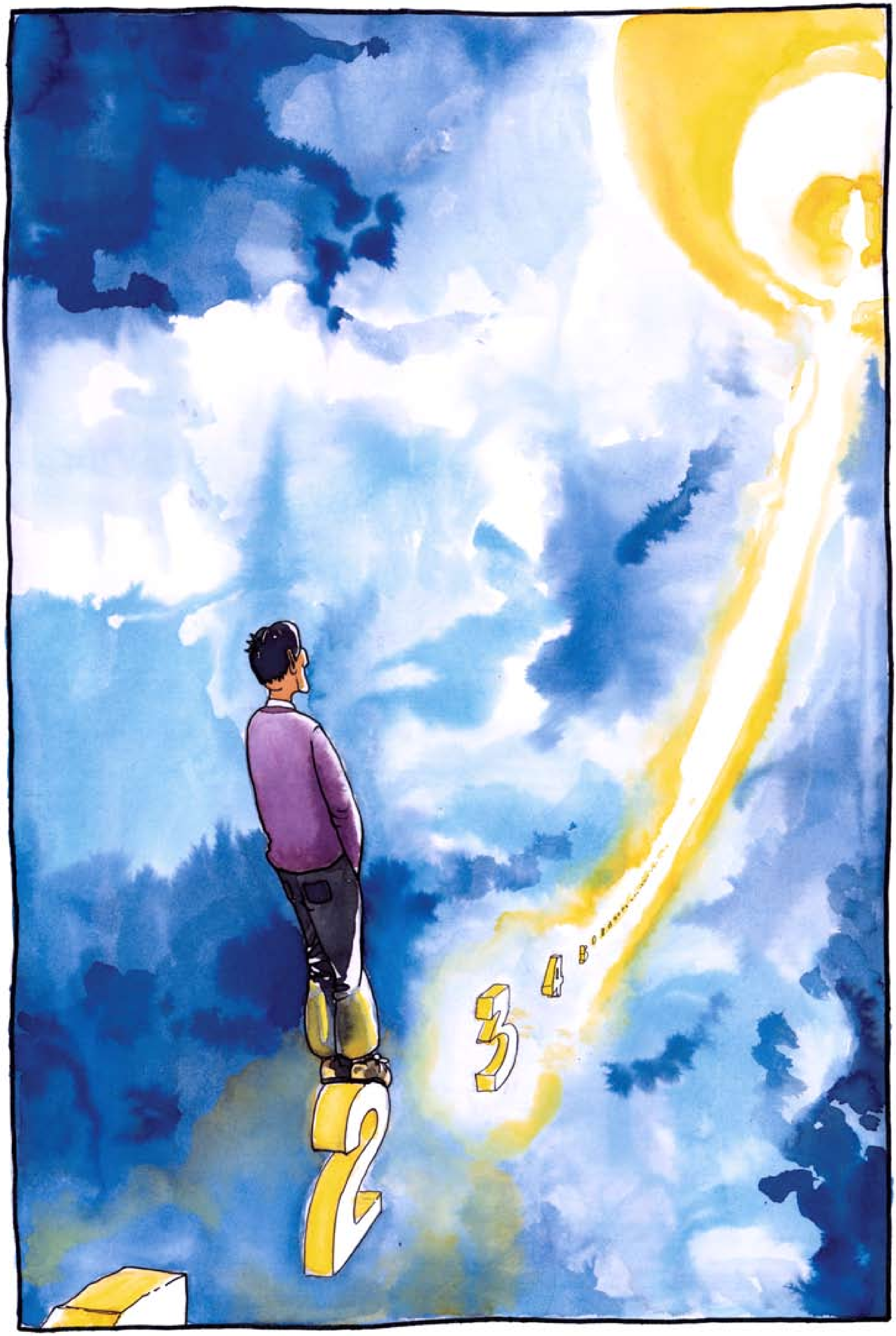

WITHIN THIS SET WE INTEND TO REPRESENT INFINITE DOTS.
TAP TAP TAP
PSSH
AND THIS, ON THE OTHER HAND, IS THE SET OF ALL THE LIVING HUMAN BEINGS ON THE EARTH.
YOU WILL AGREE THAT THERE'S NO WAY OUT.
EVEN THOUGH MANKIND GROWS AND REPRODUCES ITSELF, WE WILL NEVER BE OF AN INFINITE NUMBER.
MEN ARE, HOW CAN I PUT IT...
FINITE.
AND THIS FINITUDE PRESENTS SOME INTER-ESTING PROPERTIES WITH REGARD TO INFINITY.
AS BRILLIANTLY EXEMPLIFIED BY HILBERT'S PARADOX.

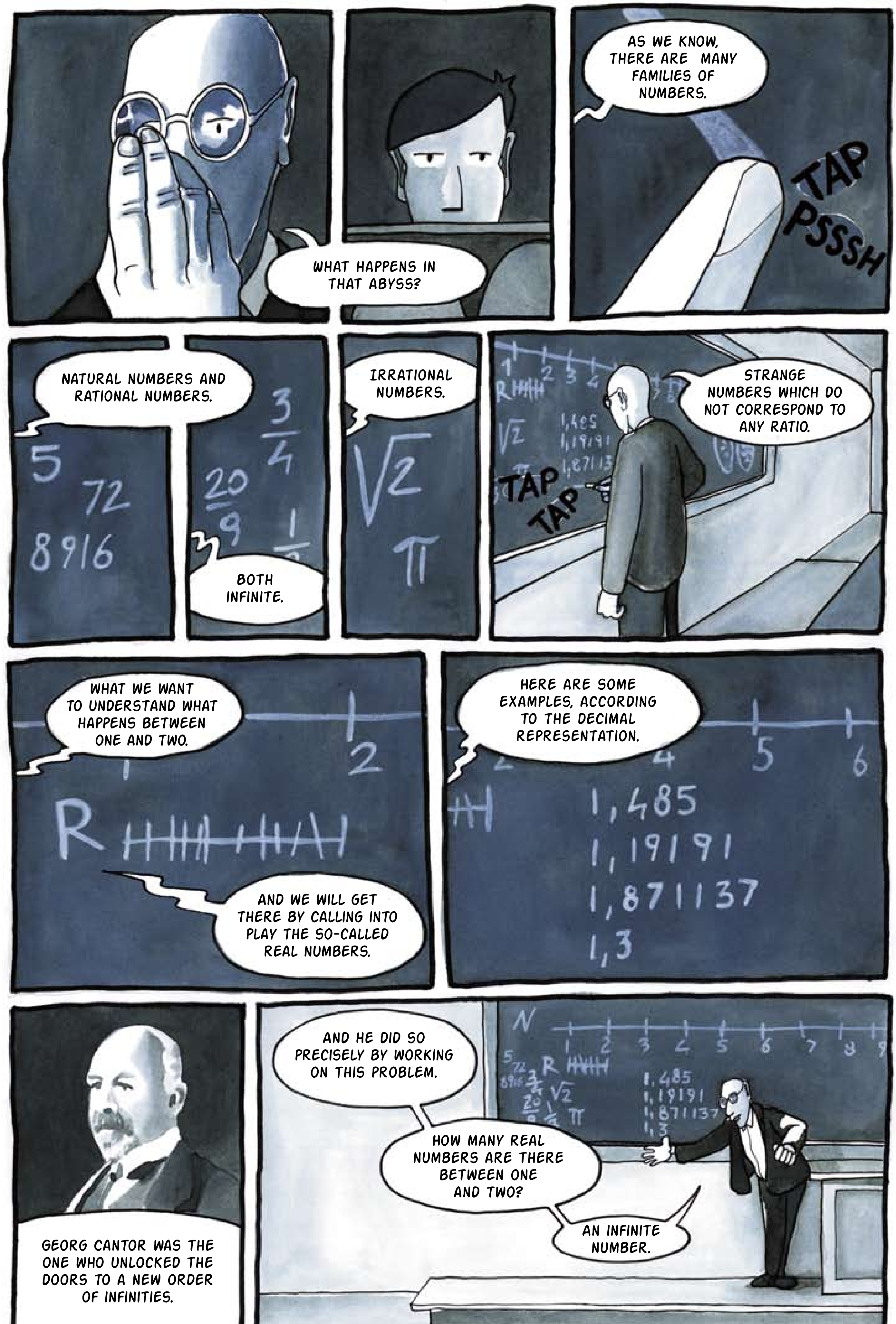

THE FAMOUS DOCTOR DAVID HILBERT...
ONE NIGHT, DOCTOR HILBERT ARRIVES AT THE HOTEL OF THE INFINITE ROOMS.
GOOD EVENING. I'D LIKE A ROOM FOR THE NIGHT.
I'M SORRY, THERE ARE NO ROOMS AVAILABLE.
WHAT DO YOU MEAN? I THOUGHT YOU HAD INFINITE ROOMS!
I'M TERRIBLY SORRY.
LISTEN TO ME. CALL THE INFINITE GUESTS OF THE HOTEL AND ASK EACH OF THEM TO MOVE TO THE ROOM RIGHT NEXT TO THEIR OWN.
SO THAT WHOEVER IS IN THE FIRST MOVES TO THE SECOND.
WHOEVER IS IN THE SECOND MOVES TO THE THIRD. WHOEVER IS IN THE THIRD MOVES TO THE FOURTH, AND SO ON.
SO THAT THE FIRST ROOM BECOMES AVAILABLE FOR USE.

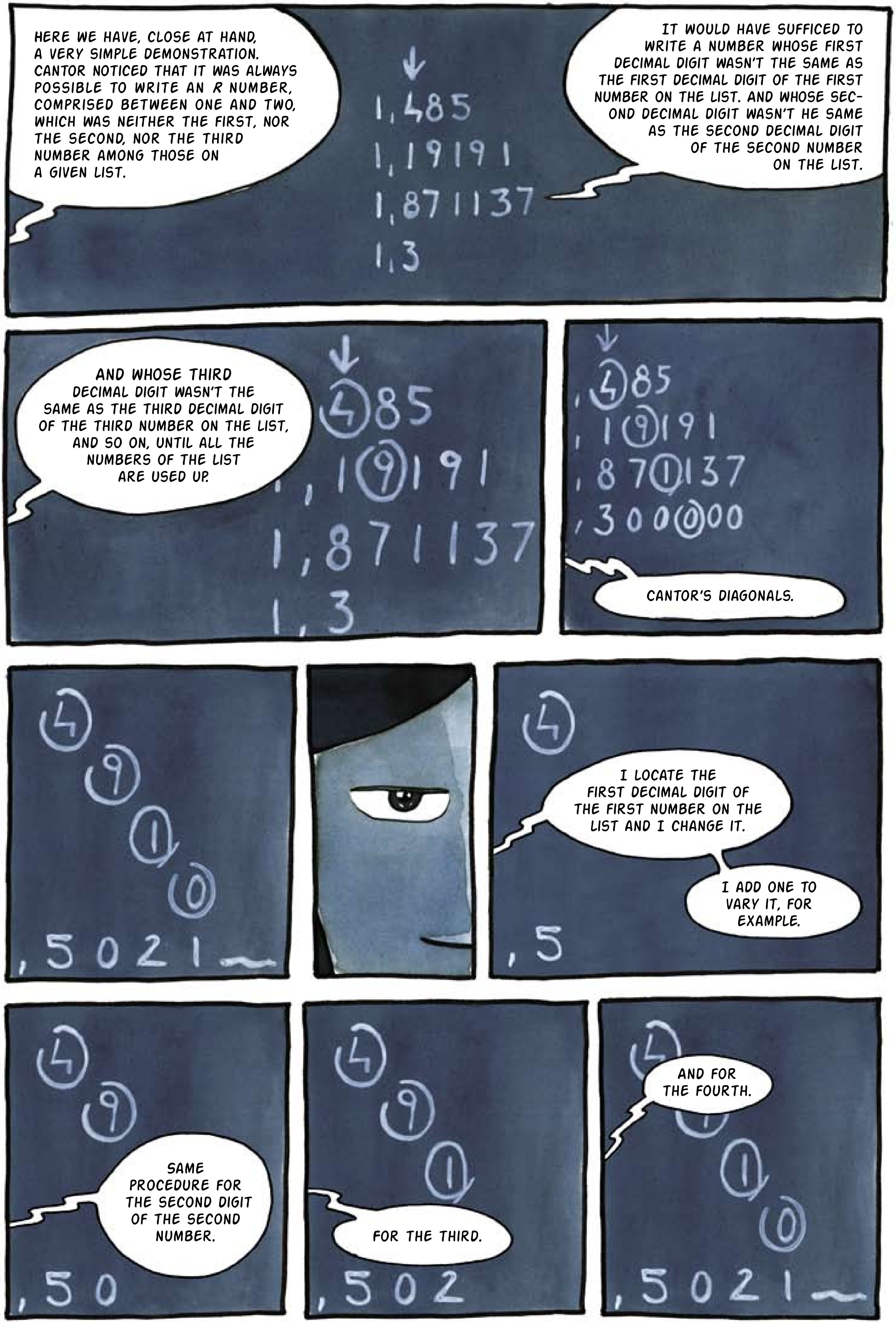

INFINITY IS SPACIOUS.
ALL NATURAL NUMBERS ARE IN IT AND STILL CAN'T FILL IT.
NO MATTER HOW BIG, THERE IS NO NATURAL NUMBER WHICH, FROM ITS HIGHEST PEAK, CAN EVEN BARELY GRAZE INFINITY.
ON THE ONE SIDE THERE IS N, RISING, AND ON THE OTHER, INFINITY.
THERE IS ALWAYS SOMETHING LIKE AN INNER TUBE SEPARATING THEM.
AND THEY WILL NEVER EVEN GRAZE EACH OTHER.


THERE IS A DISCONTINUITY IN THE MATHEMATICAL FABRIC.
IT'S AN ABSTRUSE CONCEPT. A CURIOUS FEELING.
AND AT A CLOSER LOOK, A SIMILAR DISCONTINUITY, AN INNER TUBE INFLATED WITH INEXPRESSIBILITY, CAN ALSO BE FOUND ELSEWHERE IN OUR CHAT.
N
1 2 3 4 5 6 7 8 9 10
BETWEEN ONE NUMBER AND THE NEXT, FOR EXAMPLE.
WHILE N FLIES.

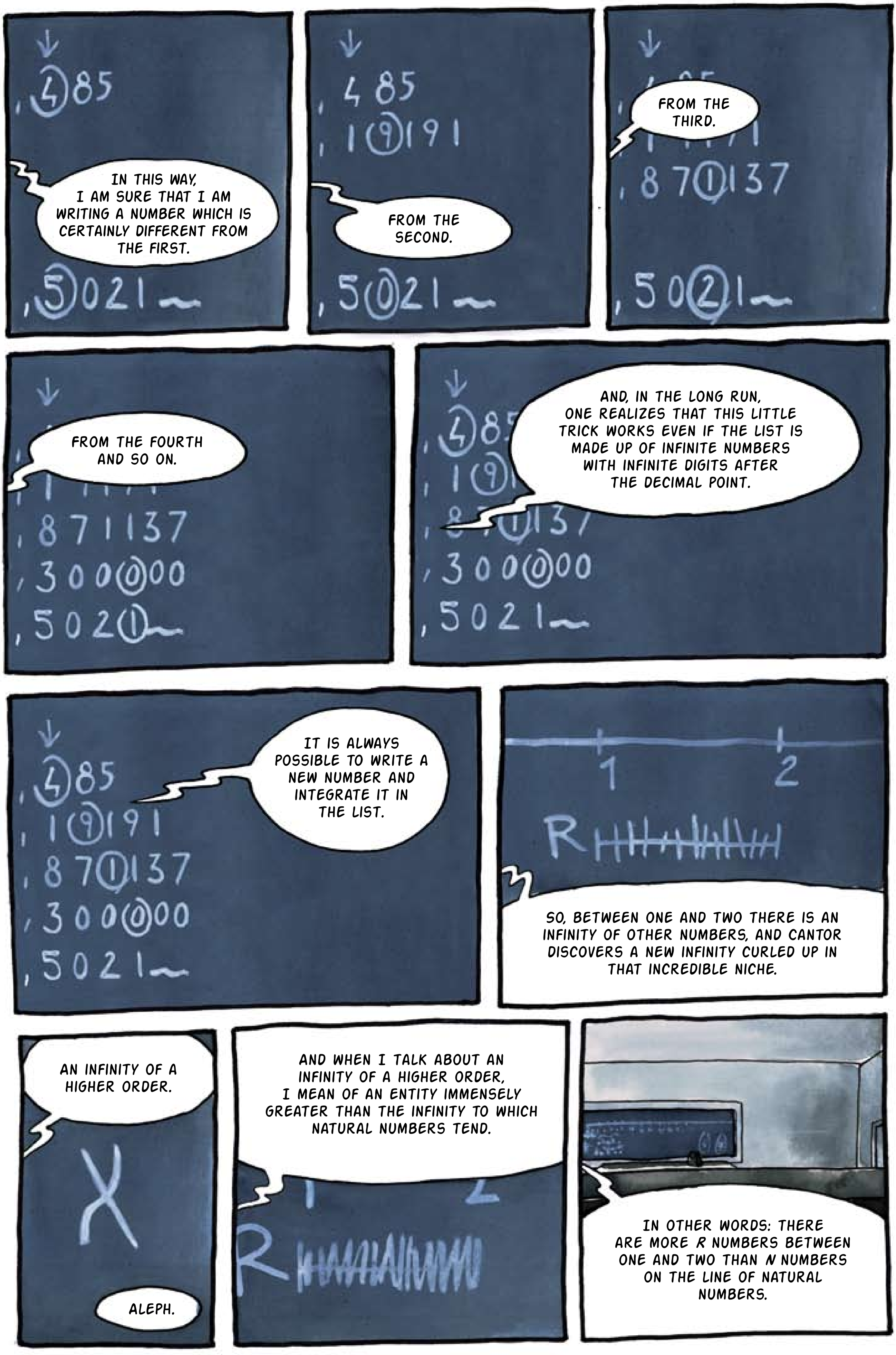

THERE IS A FRACTURE SEPARATING ONE FROM TWO.
A COMPULSORY JUMP IN THE COUNT.
FIRST ONE. JUMP. THEN TWO. JUMP. THREE. JUMP.
NOT A TRANSFORMATION OF ONE INTO TWO.
BUT ONE. AND TWO.
A MIGHTY LEAP OF NUMINOUS ODOUR - AND AN ABYSS.

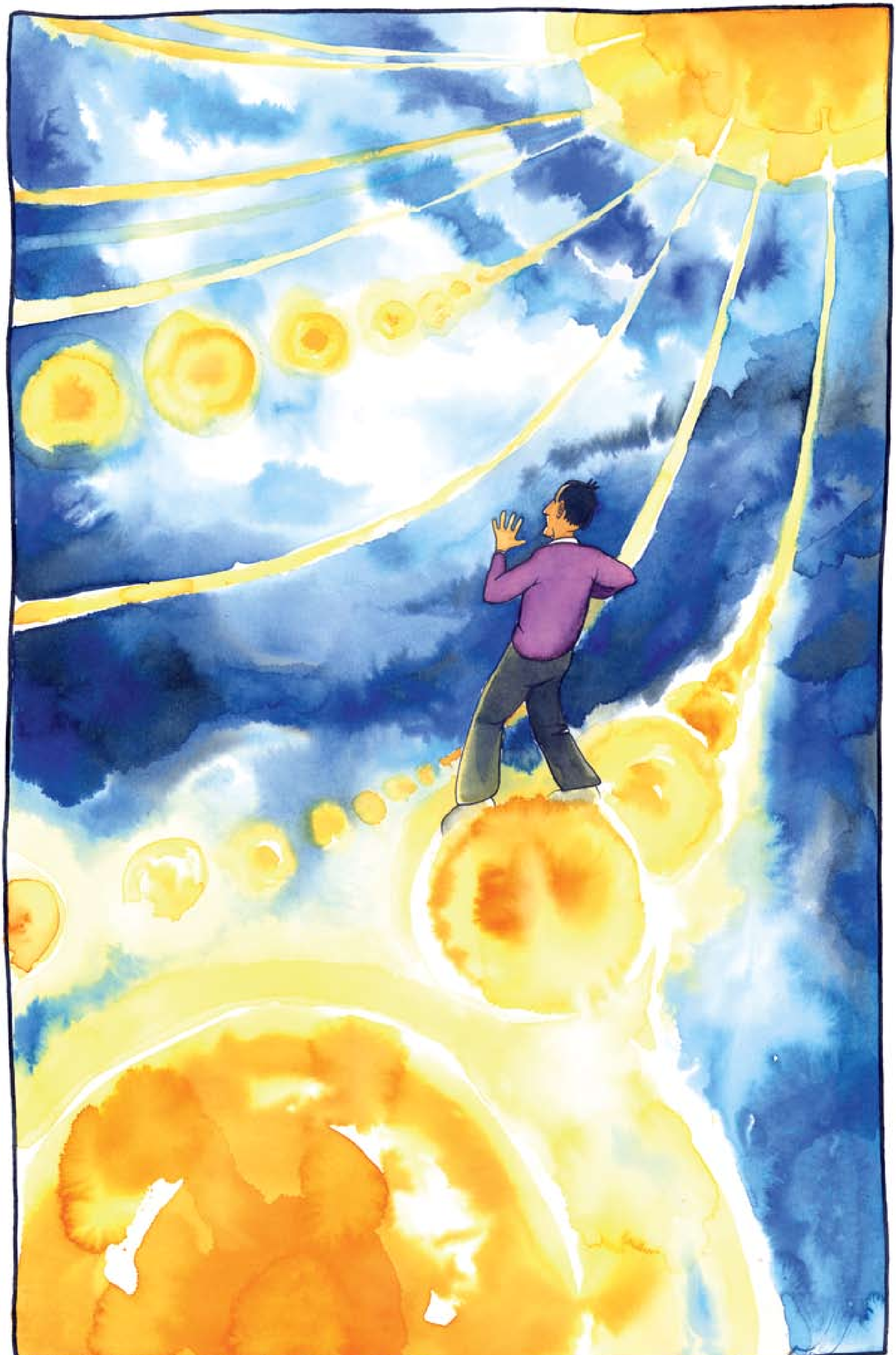

2
3
4

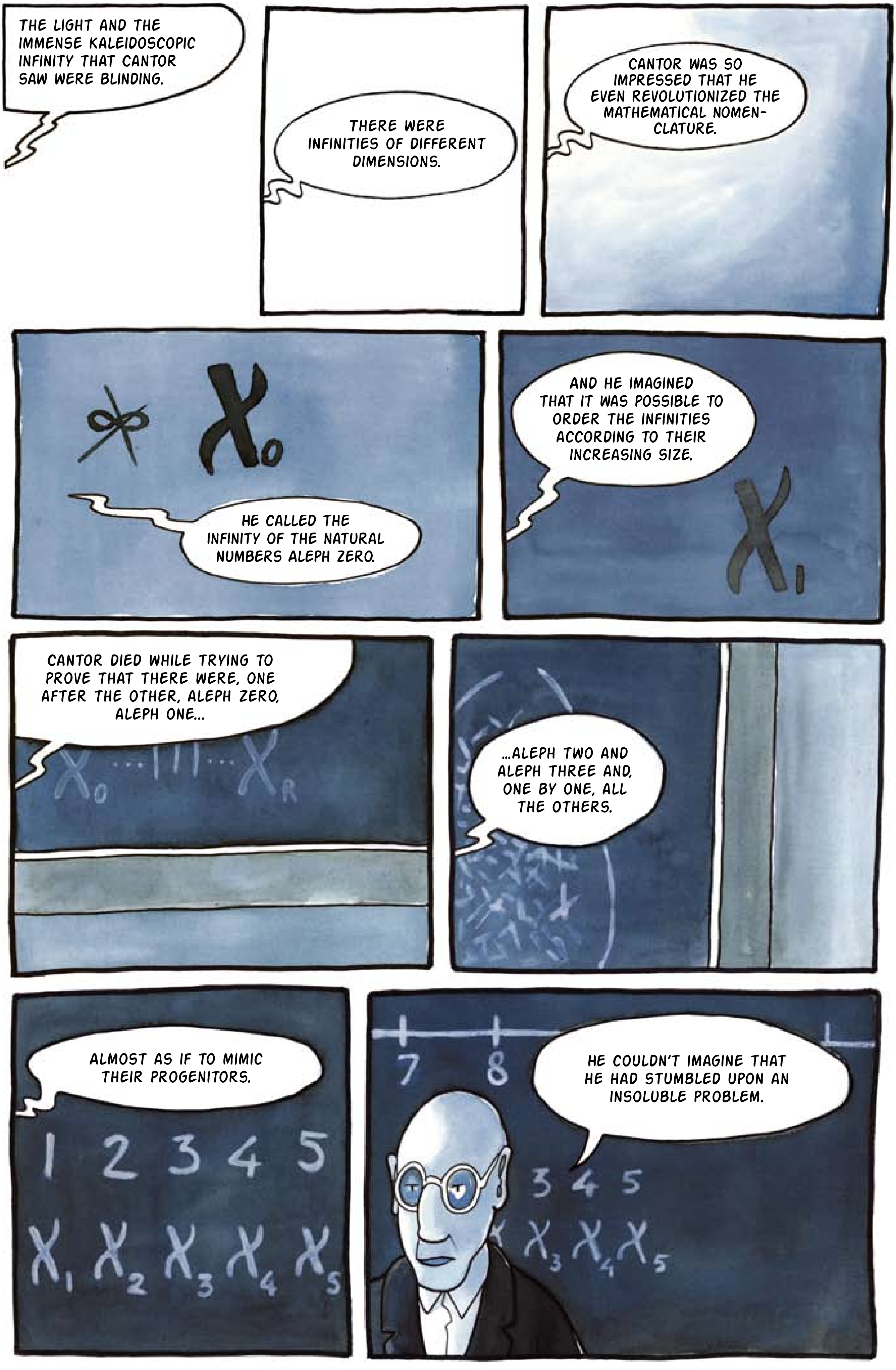

WHAT HAPPENS IN THAT ABYSS?
AS WE KNOW, THERE ARE MANY FAMILIES OF NUMBERS.
TAP
PSSSH
NATURAL NUMBERS AND RATIONAL NUMBERS.
5
72
8916
3/4
20/9
BOTH INFINITE.
IRRATIONAL NUMBERS.
√2
π
STRANGE NUMBERS WHICH DO NOT CORRESPOND TO ANY RATIO.
TAP
TAP
WHAT WE WANT TO UNDERSTAND WHAT HAPPENS BETWEEN ONE AND TWO.
2
R
AND WE WILL GET THERE BY CALLING INTO PLAY THE SO-CALLED REAL NUMBERS.
HERE ARE SOME EXAMPLES, ACCORDING TO THE DECIMAL REPRESENTATION.
4
5
6
1,485
1,19191
1,871137
1,3
GEORG CANTOR WAS THE ONE WHO UNLOCKED THE DOORS TO A NEW ORDER OF INFINITIES.
AND HE DID SO PRECISELY BY WORKING ON THIS PROBLEM.
HOW MANY REAL NUMBERS ARE THERE BETWEEN ONE AND TWO?
AN INFINITE NUMBER.

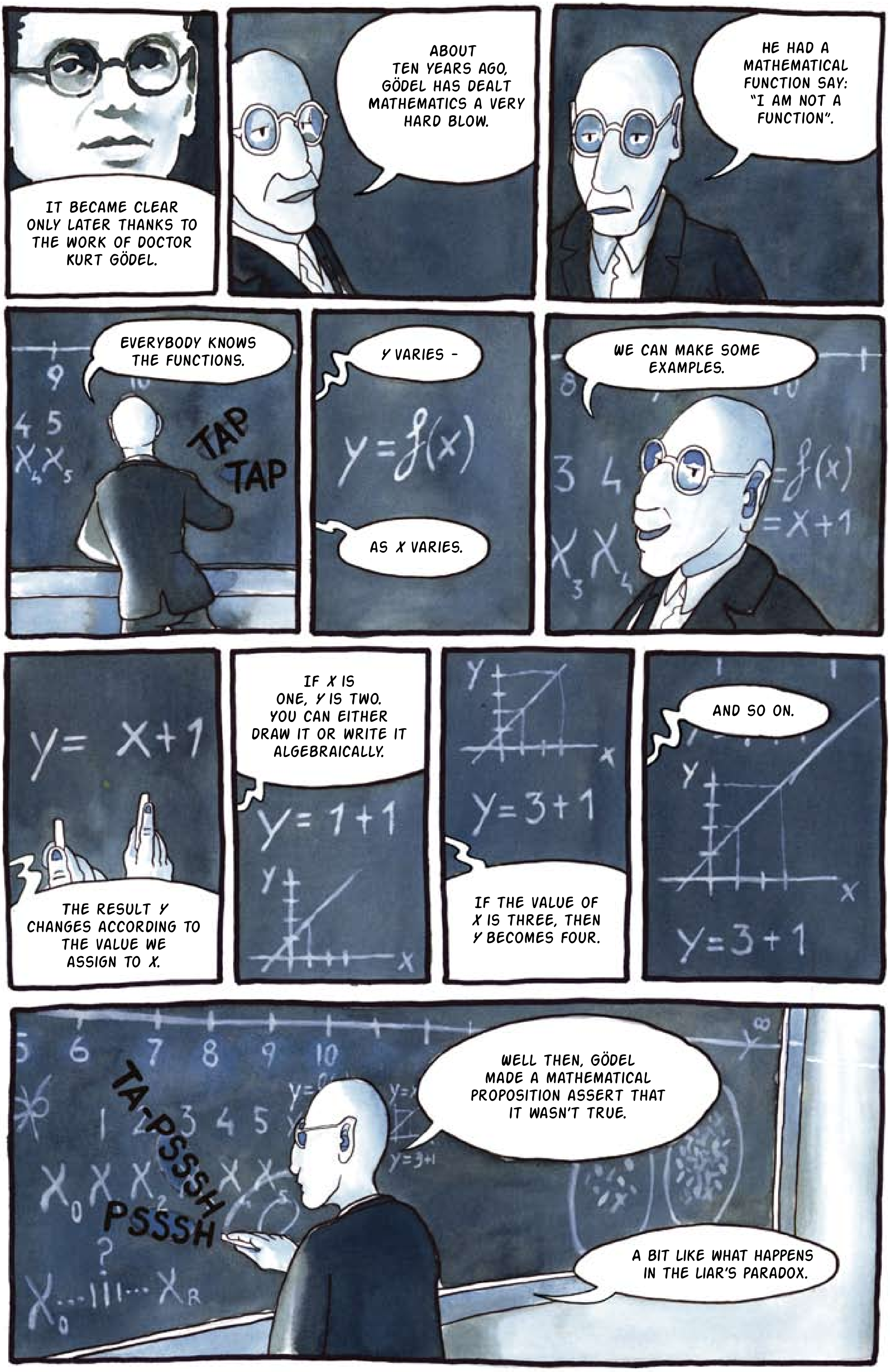


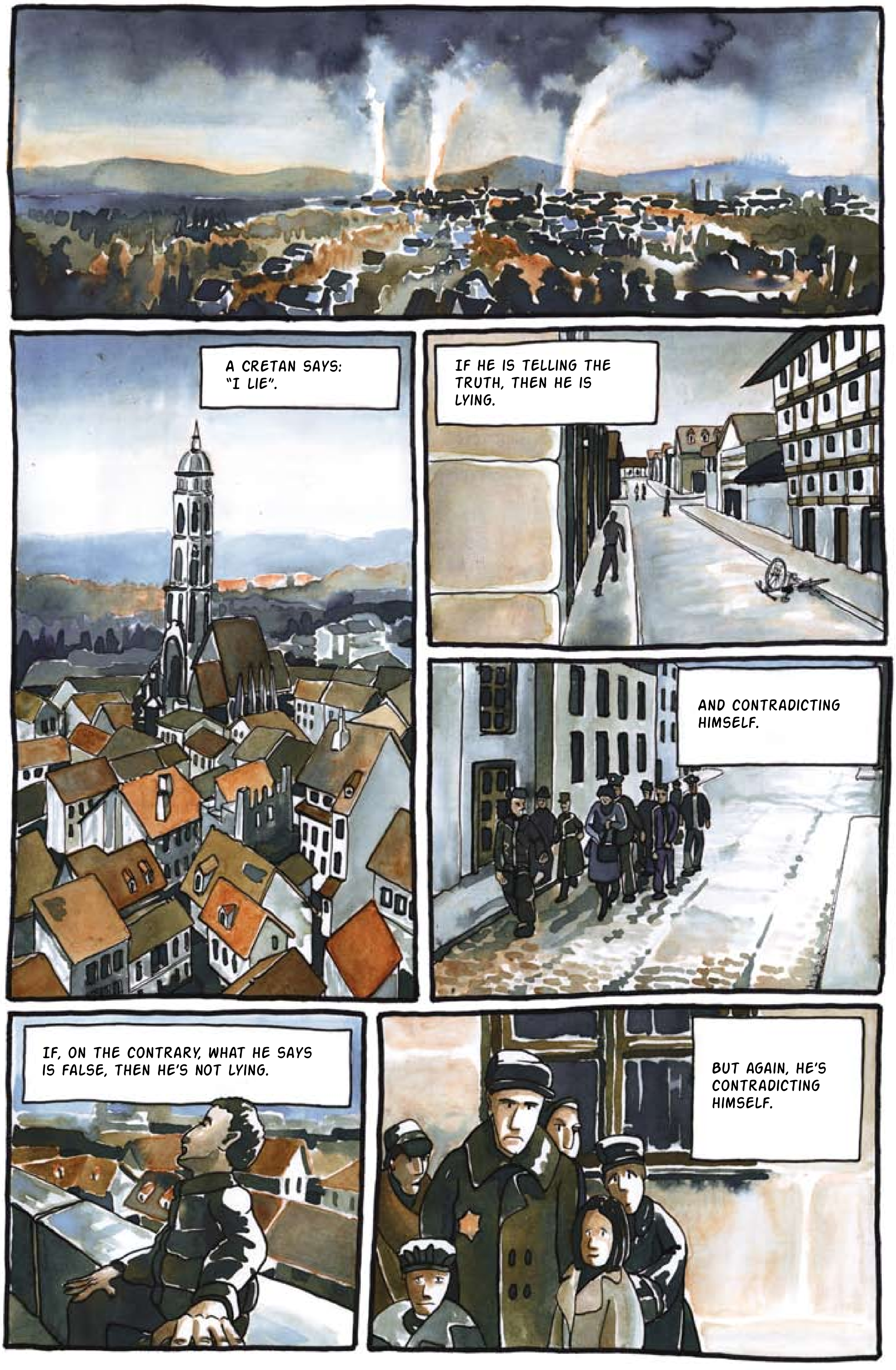
HERE WE HAVE, CLOSE AT HAND, A VERY SIMPLE DEMONSTRATION. CANTOR NOTICED THAT IT WAS ALWAYS POSSIBLE TO WRITE AN R NUMBER, COMPRISED BETWEEN ONE AND TWO, WHICH WAS NEITHER THE FIRST, NOR THE SECOND, NOR THE THIRD NUMBER AMONG THOSE ON A GIVEN LIST.
1,485
1,19191
1,871137
1,3
IT WOULD HAVE SUFFICED TO WRITE A NUMBER WHOSE FIRST DECIMAL DIGIT WASN'T THE SAME AS THE FIRST DECIMAL DIGIT OF THE FIRST NUMBER ON THE LIST. AND WHOSE SECOND DECIMAL DIGIT WASN'T HE SAME AS THE SECOND DECIMAL DIGIT OF THE SECOND NUMBER ON THE LIST.
AND WHOSE THIRD DECIMAL DIGIT WASN'T THE SAME AS THE THIRD DECIMAL DIGIT OF THE THIRD NUMBER ON THE LIST, AND SO ON, UNTIL ALL THE NUMBERS OF THE LIST ARE USED UP.
,485
,19191
,871137
,300000
CANTOR'S DIAGONALS.
,5021
I LOCATE THE FIRST DECIMAL DIGIT OF THE FIRST NUMBER ON THE LIST AND I CHANGE IT.
I ADD ONE TO VARY IT, FOR EXAMPLE.
,5
SAME PROCEDURE FOR THE SECOND DIGIT OF THE SECOND NUMBER.
,50
FOR THE THIRD.
,502
AND FOR THE FOURTH.
,5021

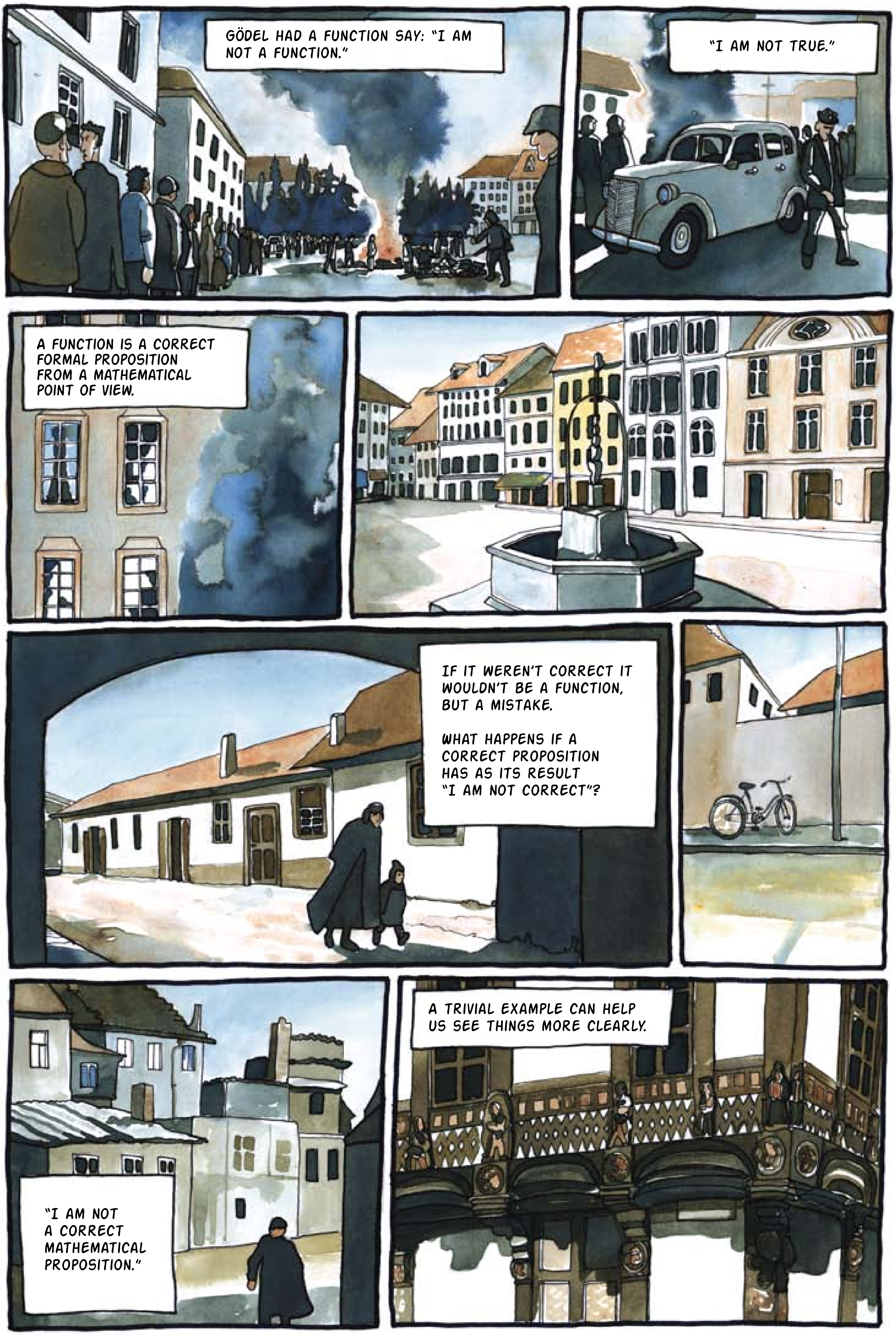

IN THIS WAY, I AM SURE THAT I AM WRITING A NUMBER WHICH IS CERTAINLY DIFFERENT FROM THE FIRST.
FROM THE SECOND.
FROM THE THIRD.
FROM THE FOURTH AND SO ON.
AND, IN THE LONG RUN, ONE REALIZES THAT THIS LITTLE TRICK WORKS EVEN IF THE LIST IS MADE UP OF INFINITE NUMBERS WITH INFINITE DIGITS AFTER THE DECIMAL POINT.
IT IS ALWAYS POSSIBLE TO WRITE A NEW NUMBER AND INTEGRATE IT IN THE LIST.
SO, BETWEEN ONE AND TWO THERE IS AN INFINITY OF OTHER NUMBERS, AND CANTOR DISCOVERS A NEW INFINITY CURLED UP IN THAT INCREDIBLE NICHE.
AN INFINITY OF A HIGHER ORDER.
ALEPH.
AND WHEN I TALK ABOUT AN INFINITY OF A HIGHER ORDER, I MEAN OF AN ENTITY IMMENSELY GREATER THAN THE INFINITY TO WHICH NATURAL NUMBERS TEND.
IN OTHER WORDS: THERE ARE MORE R NUMBERS BETWEEN ONE AND TWO THAN N NUMBERS ON THE LINE OF NATURAL NUMBERS.

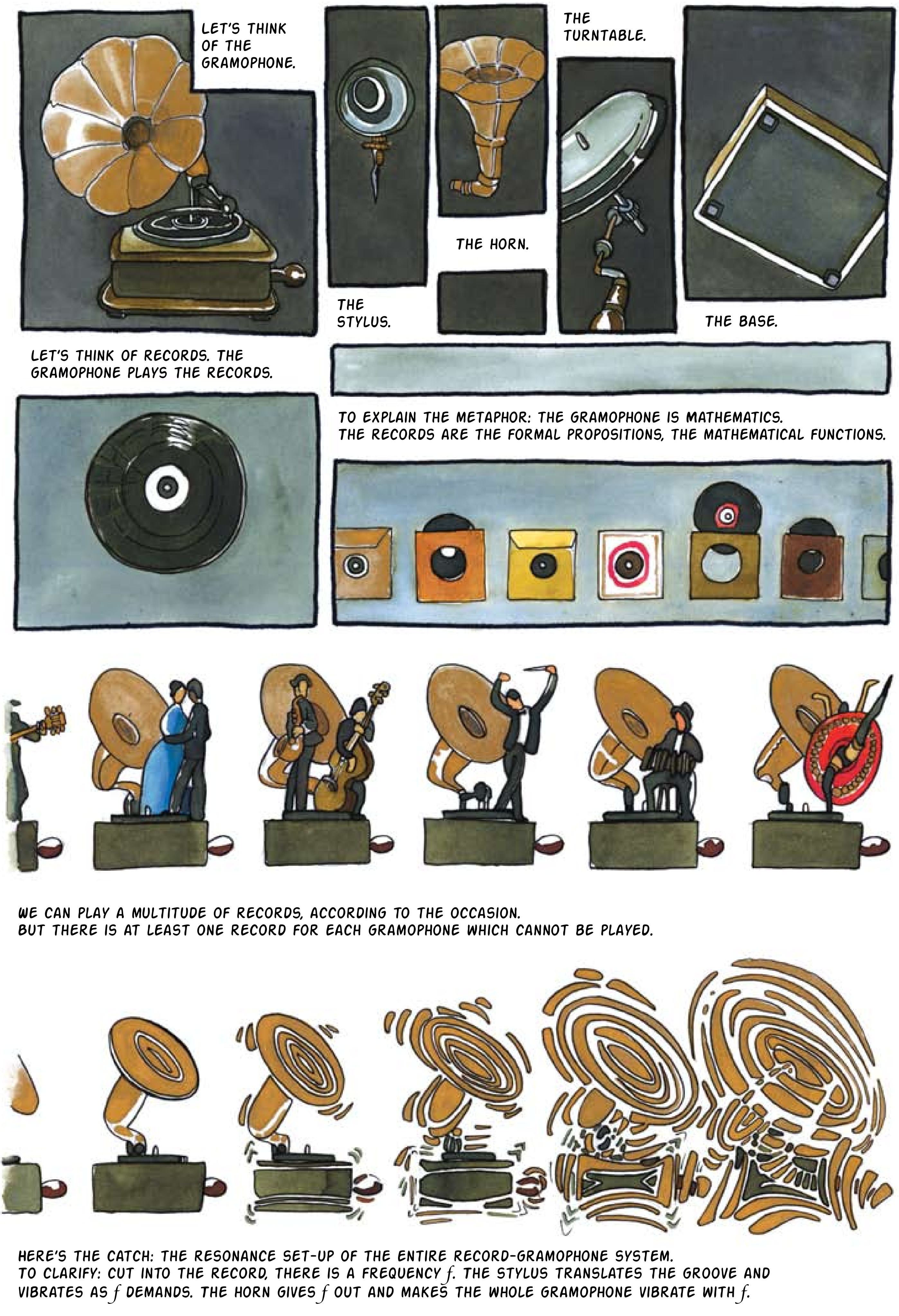



THE LIGHT AND THE IMMENSE KALEIDOSCOPIC INFINITY THAT CANTOR SAW WERE BLINDING.
THERE WERE INFINITIES OF DIFFERENT DIMENSIONS.
CANTOR WAS SO IMPRESSED THAT HE EVEN REVOLUTIONIZED THE MATHEMATICAL NOMEN-CLATURE.
HE CALLED THE INFINITY OF THE NATURAL NUMBERS ALEPH ZERO.
AND HE IMAGINED THAT IT WAS POSSIBLE TO ORDER THE INFINITIES ACCORDING TO THEIR INCREASING SIZE.
CANTOR DIED WHILE TRYING TO PROVE THAT THERE WERE, ONE AFTER THE OTHER, ALEPH ZERO, ALEPH ONE...
...ALEPH TWO AND ALEPH THREE AND, ONE BY ONE, ALL THE OTHERS.
ALMOST AS IF TO MIMIC THEIR PROGENITORS.
1 2 3 4 5
7 8
HE COULDN'T IMAGINE THAT HE HAD STUMBLED UPON AN INSOLUBLE PROBLEM.


IT BECAME CLEAR ONLY LATER THANKS TO THE WORK OF DOCTOR KURT GÖDEL.
ABOUT TEN YEARS AGO, GÖDEL HAS DEALT MATHEMATICS A VERY HARD BLOW.
HE HAD A MATHEMATICAL FUNCTION SAY: "I AM NOT A FUNCTION".
EVERYBODY KNOWS THE FUNCTIONS.
TAP
TAP
Y VARIES –
y = f(x)
AS X VARIES.
WE CAN MAKE SOME EXAMPLES.
y = X+1
THE RESULT Y CHANGES ACCORDING TO THE VALUE WE ASSIGN TO X.
IF X IS ONE, Y IS TWO. YOU CAN EITHER DRAW IT OR WRITE IT ALGEBRAICALLY.
y = 1+1
y = 3+1
IF THE VALUE OF X IS THREE, THEN Y BECOMES FOUR.
AND SO ON.
y = 3+1
WELL THEN, GÖDEL MADE A MATHEMATICAL PROPOSITION ASSERT THAT IT WASN'T TRUE.
TA-PSSSH
PSSSH
A BIT LIKE WHAT HAPPENS IN THE LIAR'S PARADOX.

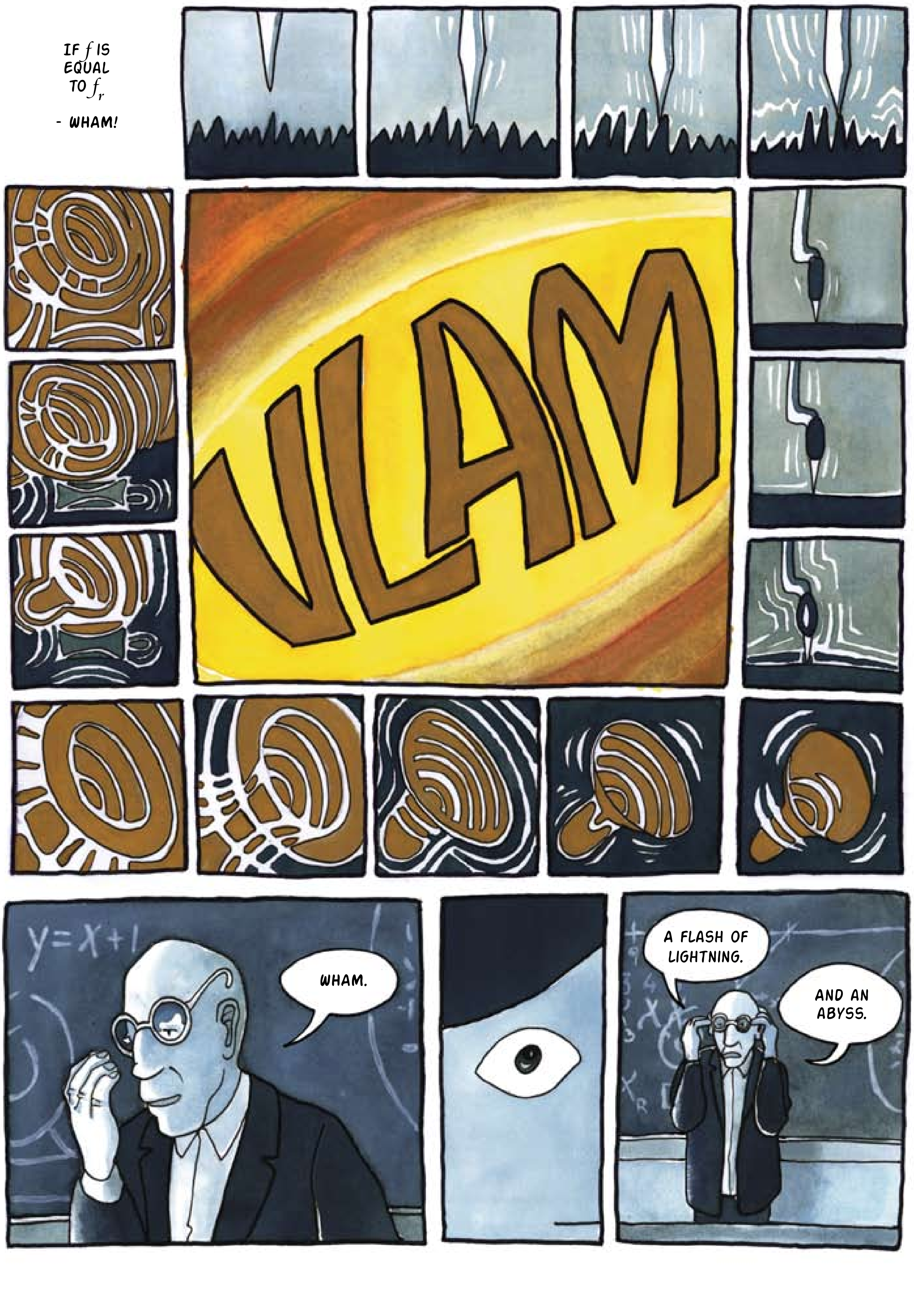

A CRETAN SAYS: "I LIE".
IF HE IS TELLING THE TRUTH, THEN HE IS LYING.
AND CONTRADICTING HIMSELF.
IF, ON THE CONTRARY, WHAT HE SAYS IS FALSE, THEN HE'S NOT LYING.
BUT AGAIN, HE'S CONTRADICTING HIMSELF.


GÖDEL HAD A FUNCTION SAY: "I AM NOT A FUNCTION."
"I AM NOT TRUE."
A FUNCTION IS A CORRECT FORMAL PROPOSITION FROM A MATHEMATICAL POINT OF VIEW.
IF IT WEREN'T CORRECT IT WOULDN'T BE A FUNCTION, BUT A MISTAKE.
WHAT HAPPENS IF A CORRECT PROPOSITION HAS AS ITS RESULT "I AM NOT CORRECT"?
"I AM NOT A CORRECT MATHEMATICAL PROPOSITION."
A TRIVIAL EXAMPLE CAN HELP US SEE THINGS MORE CLEARLY.


LET'S THINK OF THE GRAMOPHONE.
THE STYLUS.
THE HORN.
THE TURNTABLE.
THE BASE.
LET'S THINK OF RECORDS. THE GRAMOPHONE PLAYS THE RECORDS.
TO EXPLAIN THE METAPHOR: THE GRAMOPHONE IS MATHEMATICS.
THE RECORDS ARE THE FORMAL PROPOSITIONS, THE MATHEMATICAL FUNCTIONS.
WE CAN PLAY A MULTITUDE OF RECORDS, ACCORDING TO THE OCCASION.
BUT THERE IS AT LEAST ONE RECORD FOR EACH GRAMOPHONE WHICH CANNOT BE PLAYED.
HERE'S THE CATCH: THE RESONANCE SET-UP OF THE ENTIRE RECORD-GRAMOPHONE SYSTEM.
TO CLARIFY: CUT INTO THE RECORD, THERE IS A FREQUENCY f. THE STYLUS TRANSLATES THE GROOVE AND VIBRATES AS f DEMANDS. THE HORN GIVES f OUT AND MAKES THE WHOLE GRAMOPHONE VIBRATE WITH f.

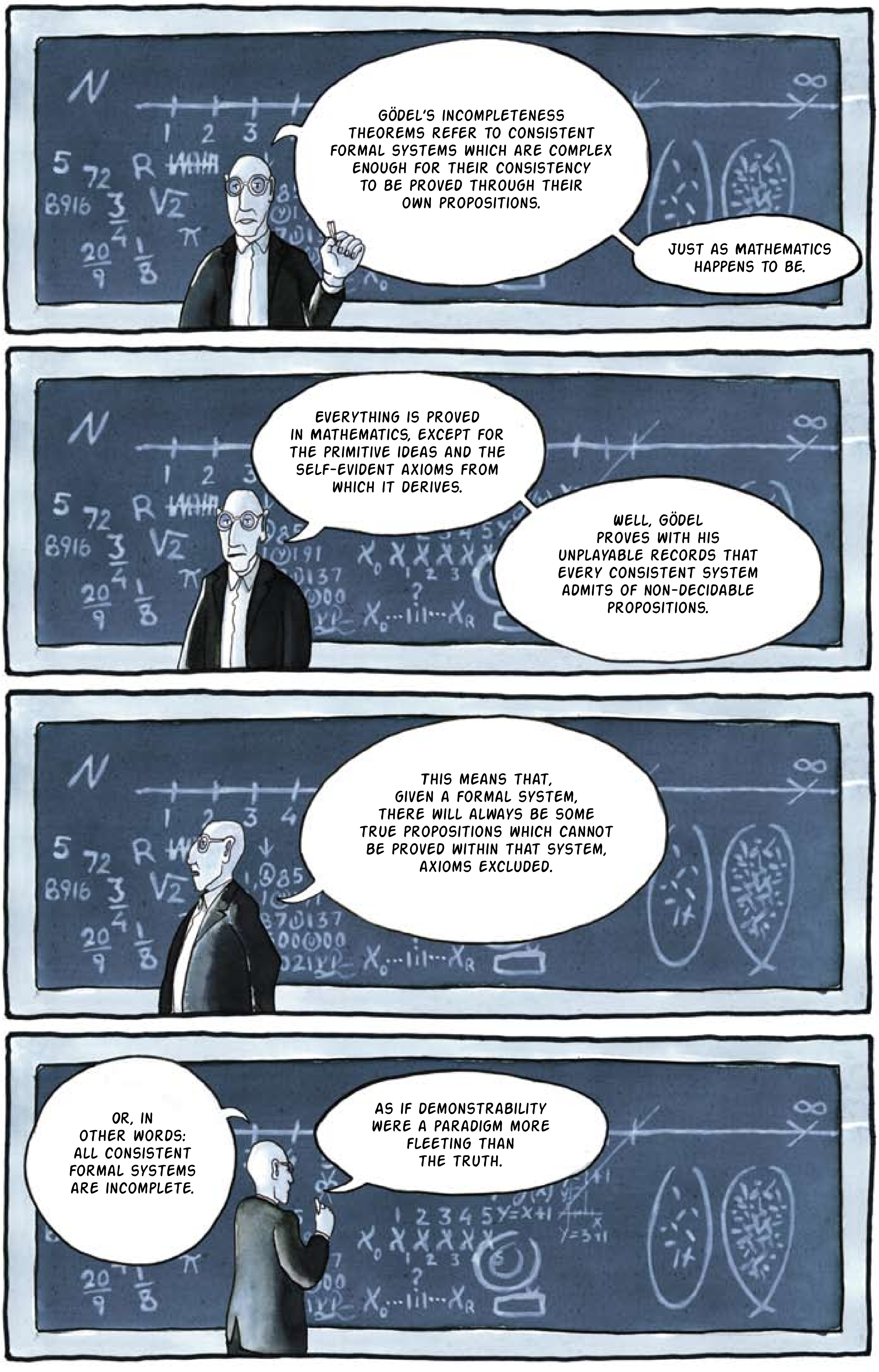
NOW, EVERY SYSTEM, BECAUSE OF ITS CONFORMATION AND MASS AND VOLUME, HAS ITS OWN PARTICULAR FREQUENCY OF RESONANCE f_r AND THE RECORD-GRAMOPHONE SYSTEM IS NO EXCEPTION.

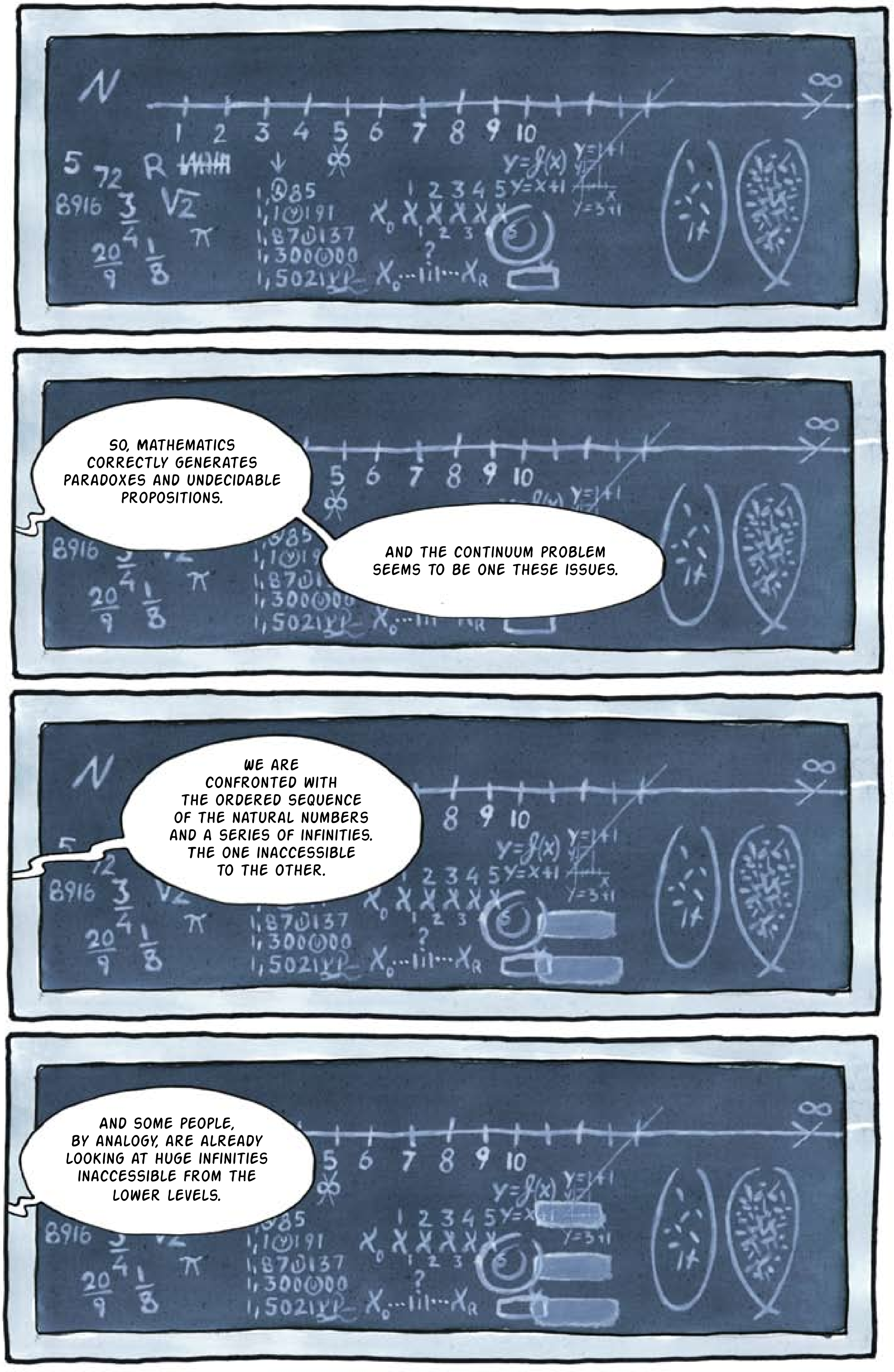
IF THE f_r FREQUENCY IS CUT INTO THE RECORD, WHAT WILL THE GRAMOPHONE PLAY?

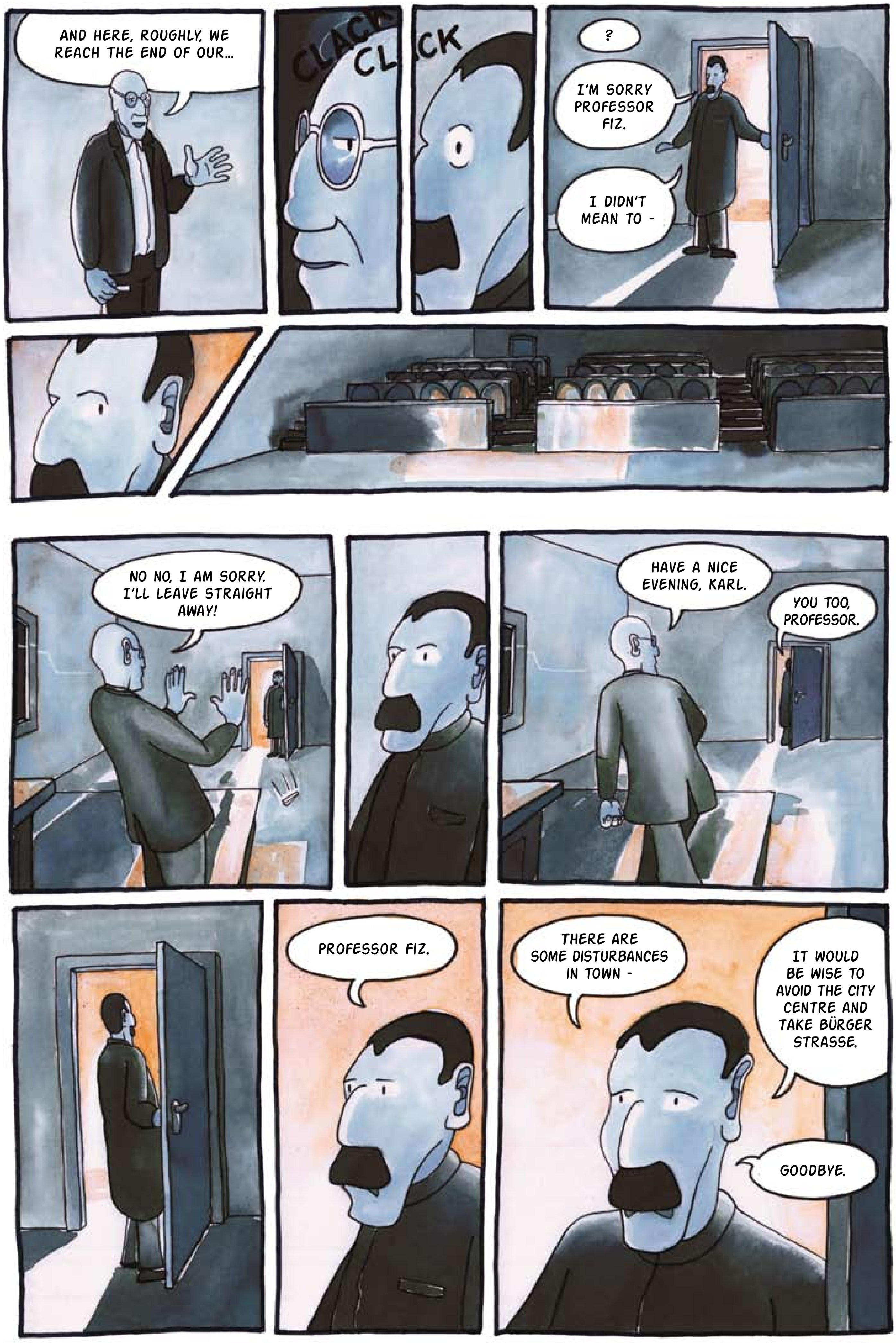


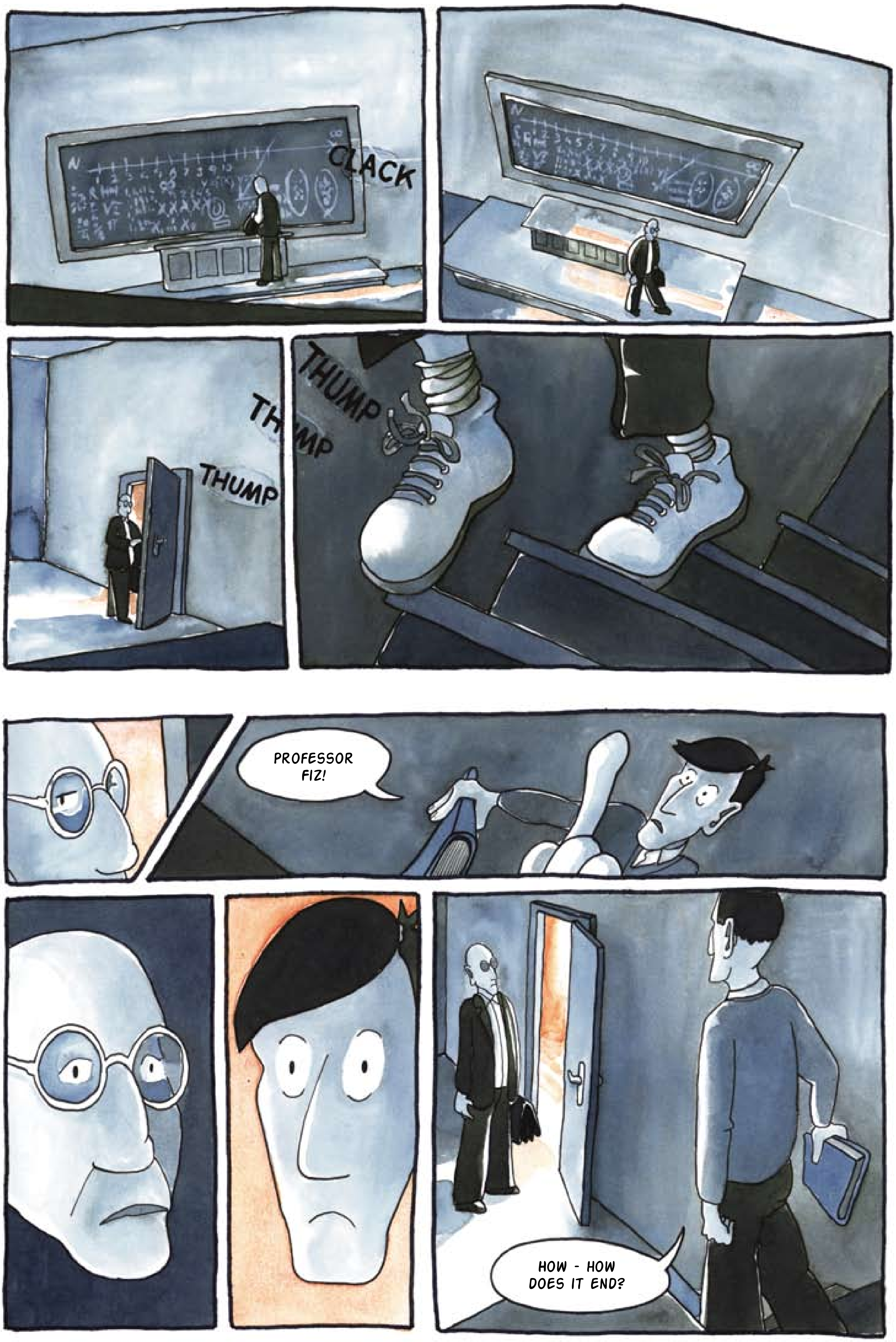

GÖDEL DISCOVERED THAT THERE WERE IN MATHEMATICS CERTAIN UNDECIDABLE, PARADOXICAL STATEMENTS, WHOSE SOLUTION COULDN'T BE ENUNCIATED.
JUST AS, IN EVERYDAY LANGUAGE, IT DOESN'T MAKE SENSE TO TRY AND MAKE OUT WHETHER THE CRETAN IS LYING OR TELLING THE TRUTH.

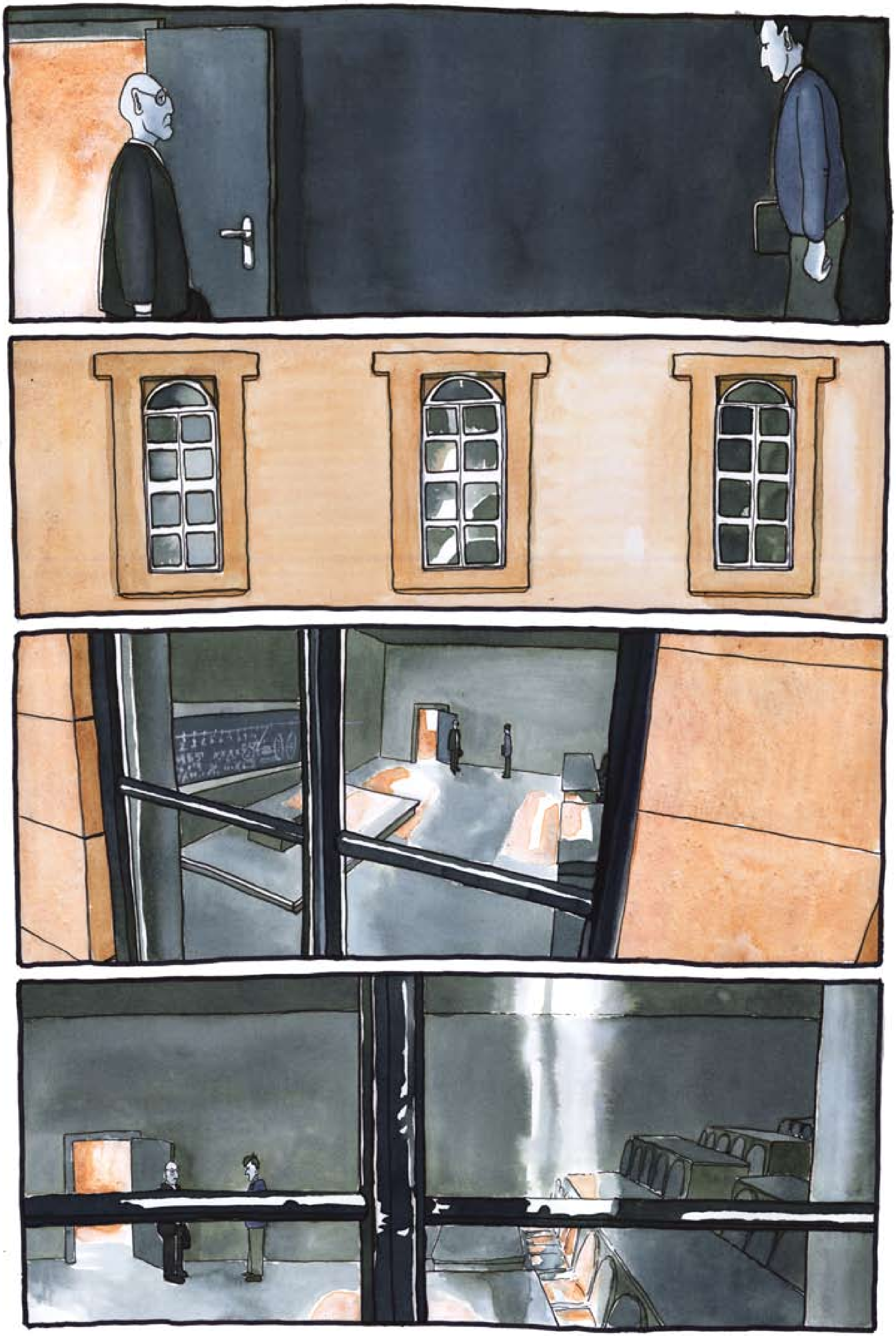

THERE ARE RECORDS WHICH, IF PLAYED, PLAY HAVOC WITH THE GRAMOPHONES.
THERE ARE MATHEMATICAL PROPORTIONS WHICH ARE A TRAP FOR MATHEMATICS.
CANTOR, UNAWARE OF THIS PARADOXICAL TRUTH, STUMBLED, UNWITTINGLY, ON AN UNDECIDABLE PROBLEM: THE CONTINUUM PROBLEM.

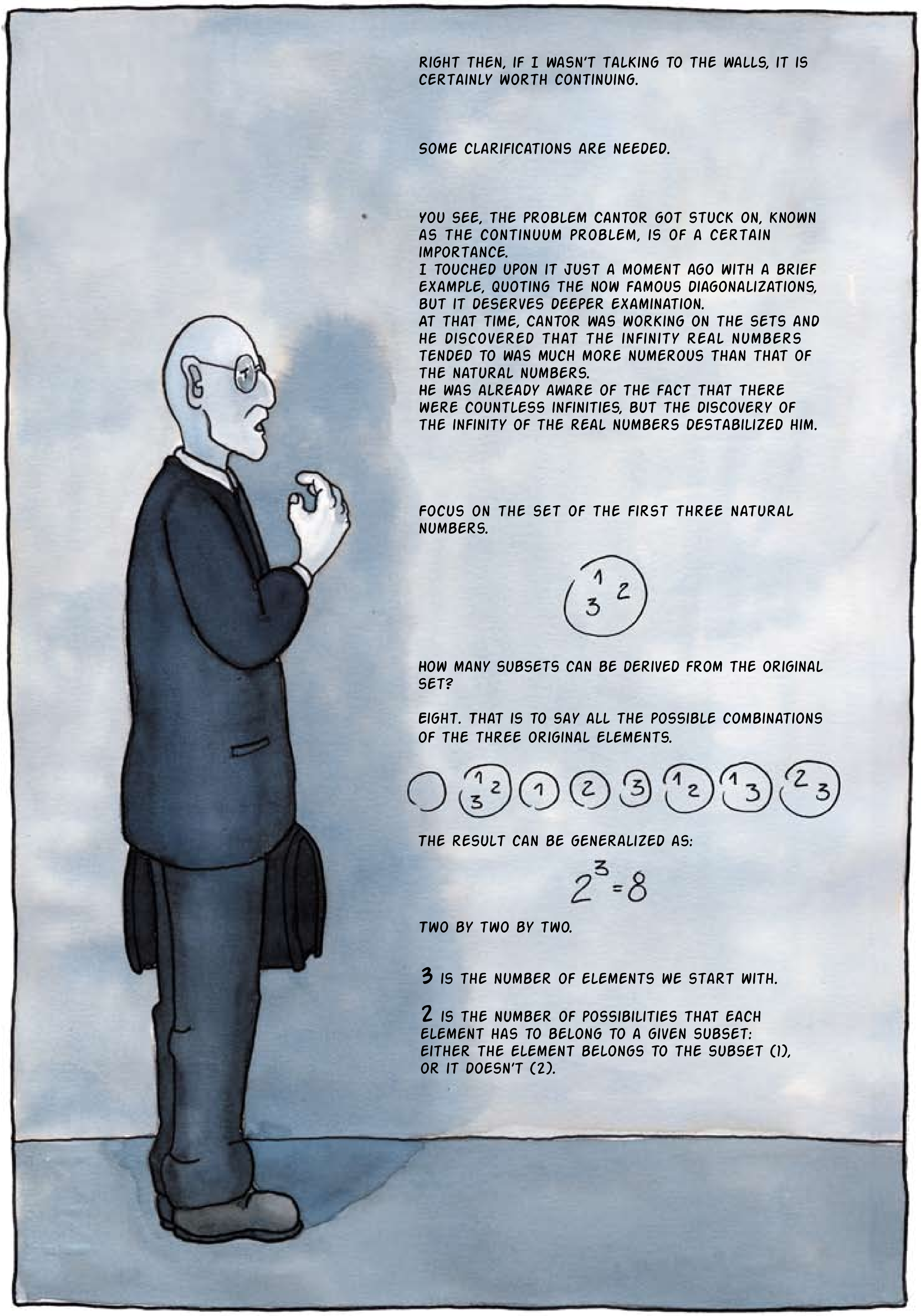

GÖDEL'S INCOMPLETENESS THEOREMS REFER TO CONSISTENT FORMAL SYSTEMS WHICH ARE COMPLEX ENOUGH FOR THEIR CONSISTENCY TO BE PROVED THROUGH THEIR OWN PROPOSITIONS.
JUST AS MATHEMATICS HAPPENS TO BE.
EVERYTHING IS PROVED IN MATHEMATICS, EXCEPT FOR THE PRIMITIVE IDEAS AND THE SELF-EVIDENT AXIOMS FROM WHICH IT DERIVES.
WELL, GÖDEL PROVES WITH HIS UNPLAYABLE RECORDS THAT EVERY CONSISTENT SYSTEM ADMITS OF NON-DECIDABLE PROPOSITIONS.
THIS MEANS THAT, GIVEN A FORMAL SYSTEM, THERE WILL ALWAYS BE SOME TRUE PROPOSITIONS WHICH CANNOT BE PROVED WITHIN THAT SYSTEM, AXIOMS EXCLUDED.
OR, IN OTHER WORDS: ALL CONSISTENT FORMAL SYSTEMS ARE INCOMPLETE.
AS IF DEMONSTRABILITY WERE A PARADIGM MORE FLEETING THAN THE TRUTH.

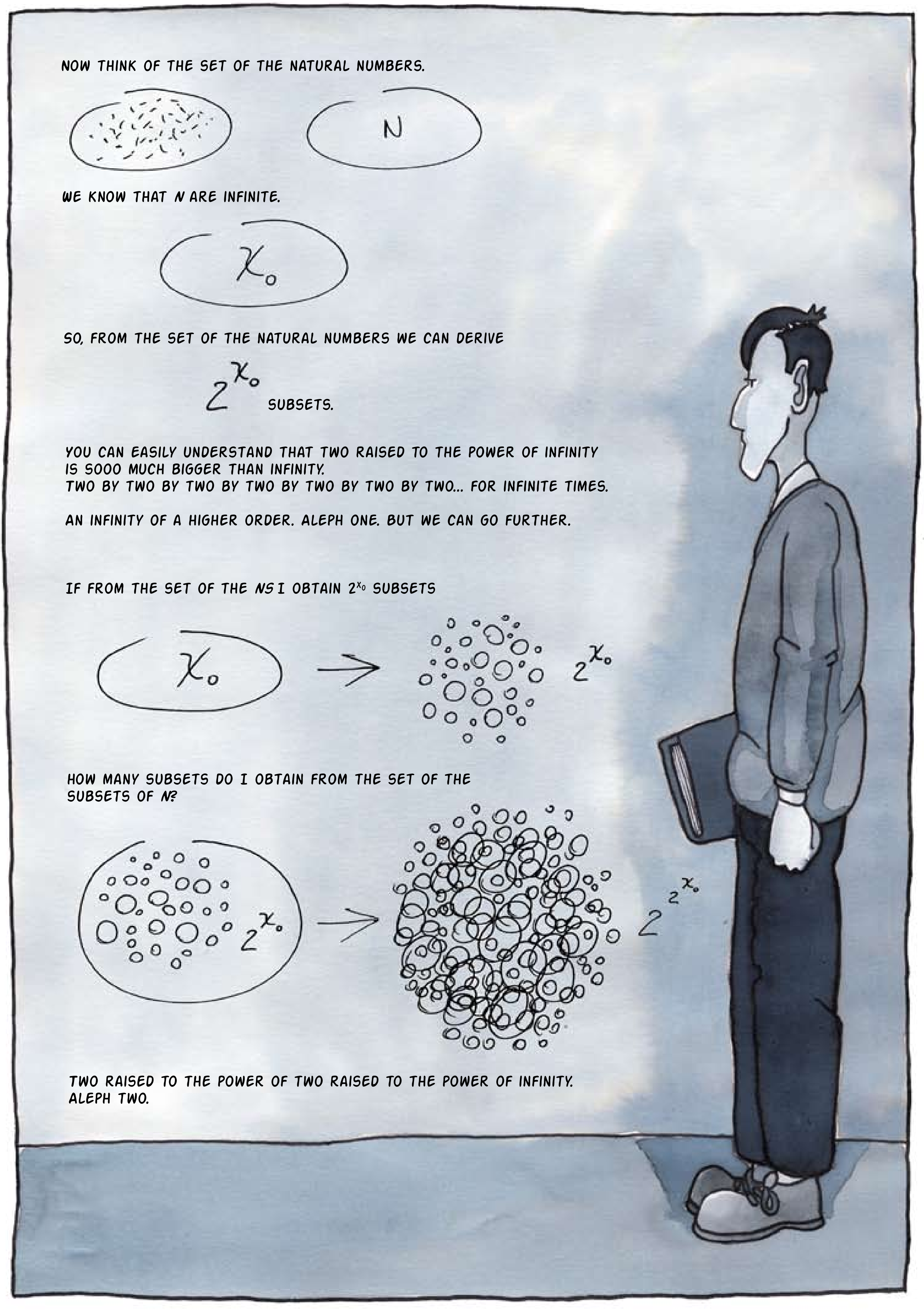

SO, MATHEMATICS CORRECTLY GENERATES PARADOXES AND UNDECIDABLE PROPOSITIONS.
AND THE CONTINUUM PROBLEM SEEMS TO BE ONE THESE ISSUES.
WE ARE CONFRONTED WITH THE ORDERED SEQUENCE OF THE NATURAL NUMBERS AND A SERIES OF INFINITIES. THE ONE INACCESSIBLE TO THE OTHER.
AND SOME PEOPLE, BY ANALOGY, ARE ALREADY LOOKING AT HUGE INFINITIES INACCESSIBLE FROM THE LOWER LEVELS.


AND HERE, ROUGHLY, WE REACH THE END OF OUR...
CLACK
CLACK
?
I'M SORRY PROFESSOR FIZ.
I DIDN'T MEAN TO -
NO NO, I AM SORRY. I'LL LEAVE STRAIGHT AWAY!
HAVE A NICE EVENING, KARL.
YOU TOO, PROFESSOR.
PROFESSOR FIZ.
THERE ARE SOME DISTURBANCES IN TOWN -
IT WOULD BE WISE TO AVOID THE CITY CENTRE AND TAKE BÜRGER STRASSE.
GOODBYE.

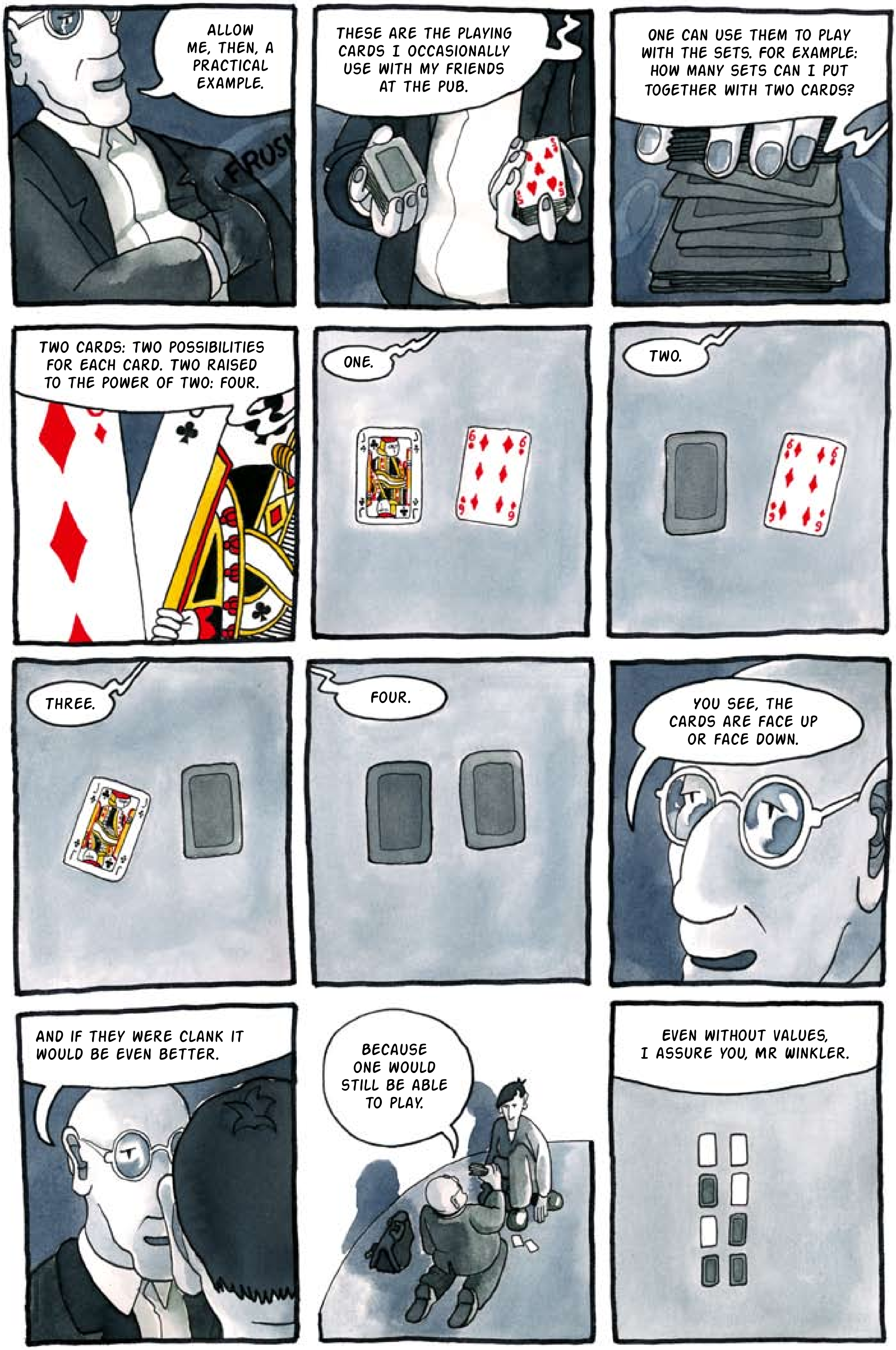

CLACK
THUMP
THUMP
THUMP
PROFESSOR FIZ!
HOW - HOW DOES IT END?



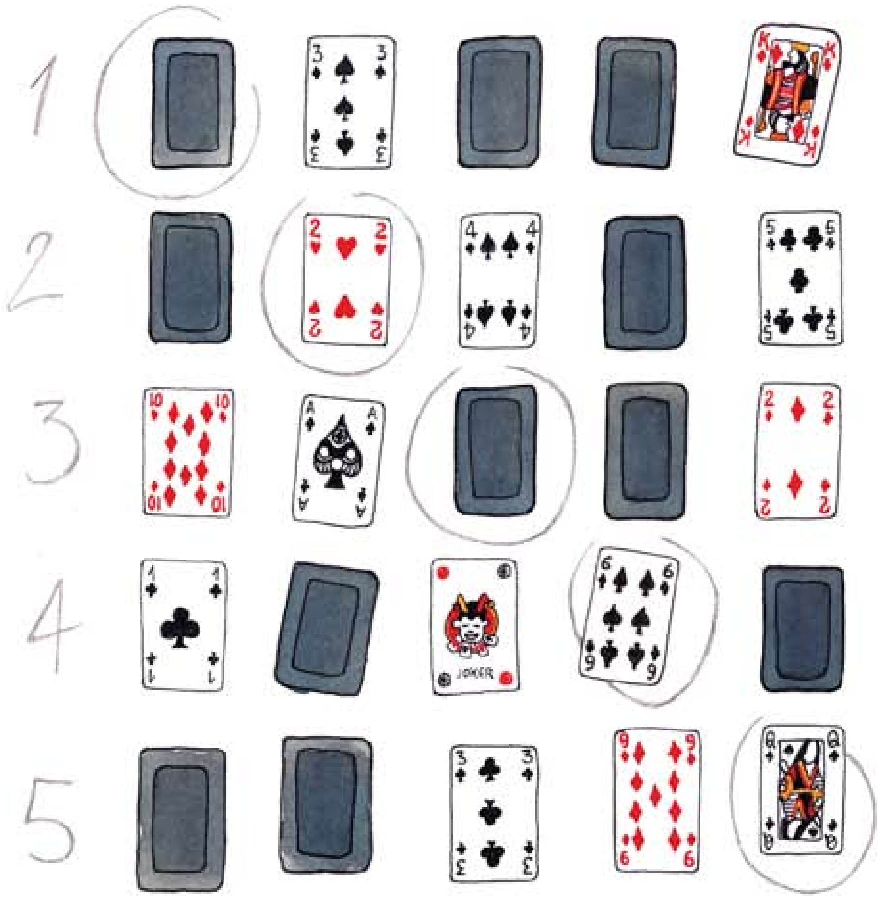

RIGHT THEN, IF I WASN'T TALKING TO THE WALLS, IT IS CERTAINLY WORTH CONTINUING.
SOME CLARIFICATIONS ARE NEEDED.
YOU SEE, THE PROBLEM CANTOR GOT STUCK ON, KNOWN AS THE CONTINUUM PROBLEM, IS OF A CERTAIN IMPORTANCE.
I TOUCHED UPON IT JUST A MOMENT AGO WITH A BRIEF EXAMPLE, QUOTING THE NOW FAMOUS DIAGONALIZATIONS, BUT IT DESERVES DEEPER EXAMINATION.
AT THAT TIME, CANTOR WAS WORKING ON THE SETS AND HE DISCOVERED THAT THE INFINITY REAL NUMBERS TENDED TO WAS MUCH MORE NUMEROUS THAN THAT OF THE NATURAL NUMBERS.
HE WAS ALREADY AWARE OF THE FACT THAT THERE WERE COUNTLESS INFINITIES, BUT THE DISCOVERY OF THE INFINITY OF THE REAL NUMBERS DESTABILIZED HIM.
FOCUS ON THE SET OF THE FIRST THREE NATURAL NUMBERS.
1 2 3
HOW MANY SUBSETS CAN BE DERIVED FROM THE ORIGINAL SET?
EIGHT. THAT IS TO SAY ALL THE POSSIBLE COMBINATIONS OF THE THREE ORIGINAL ELEMENTS.
THE RESULT CAN BE GENERALIZED AS:
$2^3=8$
TWO BY TWO BY TWO.
3 IS THE NUMBER OF ELEMENTS WE START WITH.
2 IS THE NUMBER OF POSSIBILITIES THAT EACH ELEMENT HAS TO BELONG TO A GIVEN SUBSET: EITHER THE ELEMENT BELONGS TO THE SUBSET (1), OR IT DOESN'T (2).

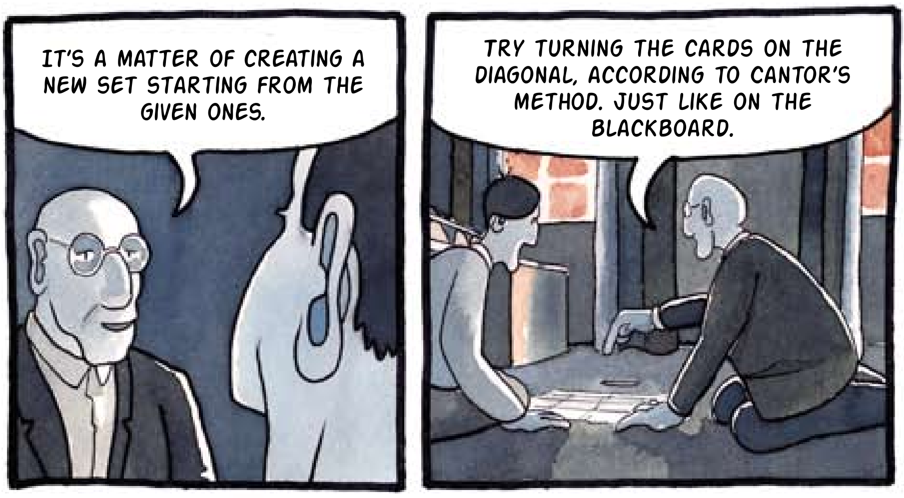

NOW THINK OF THE SET OF THE NATURAL NUMBERS.
N
WE KNOW THAT N ARE INFINITE.
$\aleph_0$
SO, FROM THE SET OF THE NATURAL NUMBERS WE CAN DERIVE
$2^{\aleph_0}$ SUBSETS.
YOU CAN EASILY UNDERSTAND THAT TWO RAISED TO THE POWER OF INFINITY
IS SOOO MUCH BIGGER THAN INFINITY.
TWO BY TWO BY TWO BY TWO BY TWO BY TWO BY TWO... FOR INFINITE TIMES.
AN INFINITY OF A HIGHER ORDER. ALEPH ONE. BUT WE CAN GO FURTHER.
IF FROM THE SET OF THE NS I OBTAIN $2^{\aleph_0}$ SUBSETS
$\aleph_0$
$2^{\aleph_0}$
HOW MANY SUBSETS DO I OBTAIN FROM THE SET OF THE
SUBSETS OF N?
$2^{\aleph_0}$
$2^{2^{\aleph_0}}$
TWO RAISED TO THE POWER OF TWO RAISED TO THE POWER OF INFINITY.
ALEPH TWO.

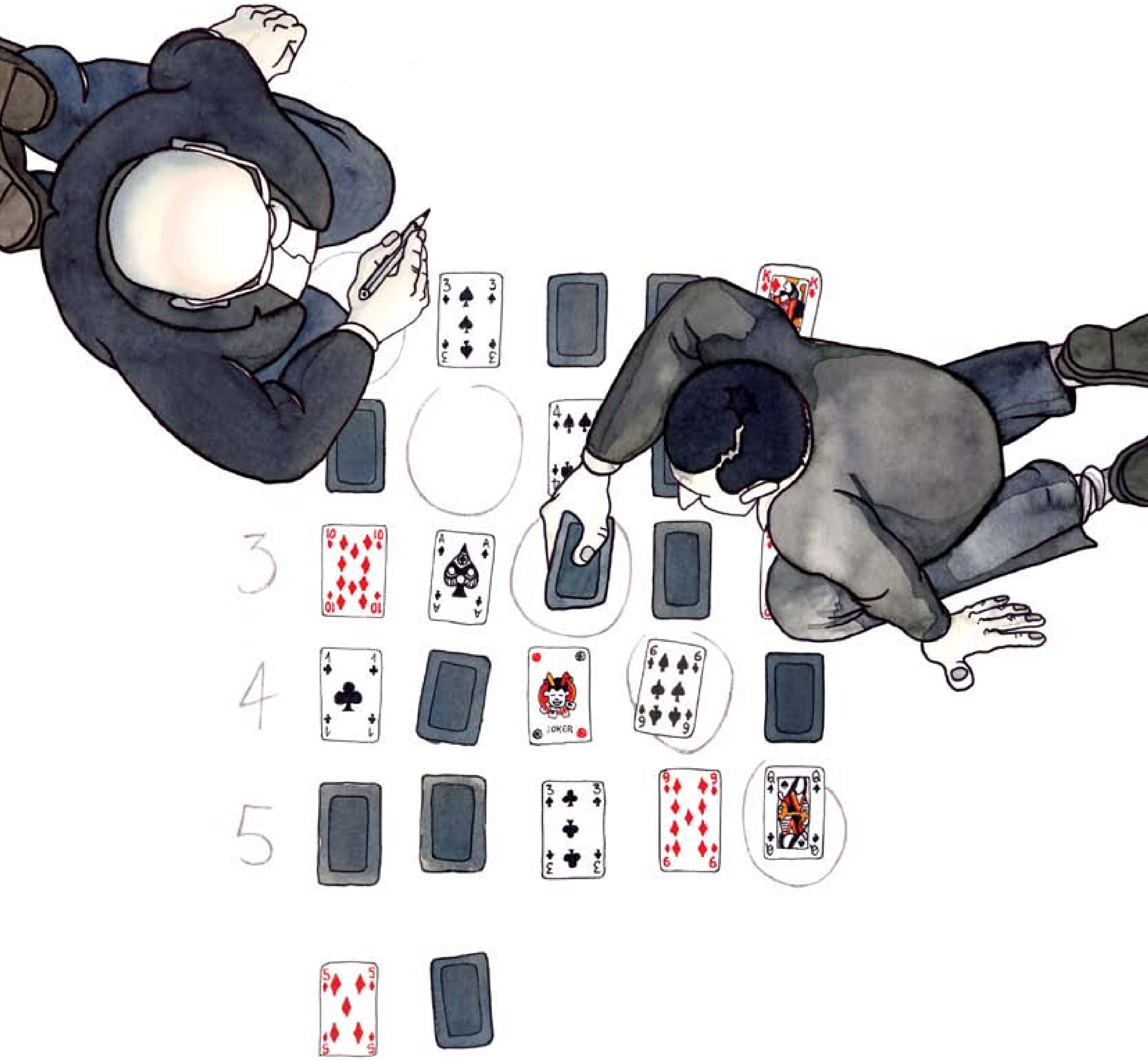

HOLD ON PROFESSOR, I'M A BIT LOST.
EXCUSE ME, YOU ARE MISTER...?
WINKLER. ALKUIN WINKLER. STUDENT AT THE FACULTY OF MATHEMATICS HERE IN GÖTTINGEN!
WELL, MR WINKLER. WHAT IS IT YOU HAVEN'T UNDERSTOOD SO FAR?
I HAVE UNDERSTOOD THAT IN MATHEMATICS THERE ARE MYRIADS OF INFINITIES OF DIFFERENT SIZES.
INFINITE INFINITIES.
1 2 3 4 5
$\aleph_0$ $\aleph_1$ $\aleph_2$ $\aleph_3$ $\aleph_4$
WHOSE PROGRESSION IS SIMILAR TO THAT OF THE NUMBERS BUT, HOW CAN I PUT IT –
WHAT DO CANTOR'S DIAGONALS HAVE TO DO WITH IT?

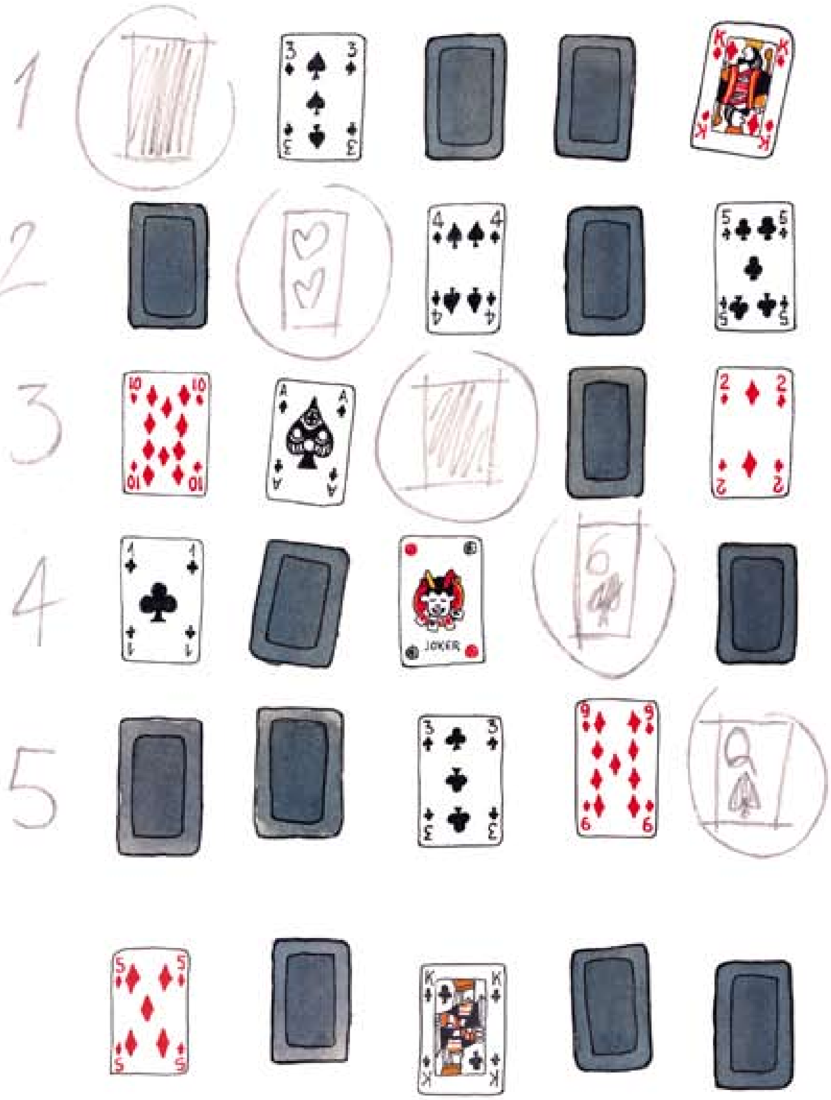

ALLOW ME, THEN, A PRACTICAL EXAMPLE.
THESE ARE THE PLAYING CARDS I OCCASIONALLY USE WITH MY FRIENDS AT THE PUB.
ONE CAN USE THEM TO PLAY WITH THE SETS. FOR EXAMPLE: HOW MANY SETS CAN I PUT TOGETHER WITH TWO CARDS?
TWO CARDS: TWO POSSIBILITIES FOR EACH CARD. TWO RAISED TO THE POWER OF TWO: FOUR.
ONE.
TWO.
THREE.
FOUR.
YOU SEE, THE CARDS ARE FACE UP OR FACE DOWN.
AND IF THEY WERE CLANK IT WOULD BE EVEN BETTER.
BECAUSE ONE WOULD STILL BE ABLE TO PLAY.
EVEN WITHOUT VALUES, I ASSURE YOU, MR WINKLER.


NOW LET'S PUT TOGETHER SOME RANDOM SETS.
PSSSH

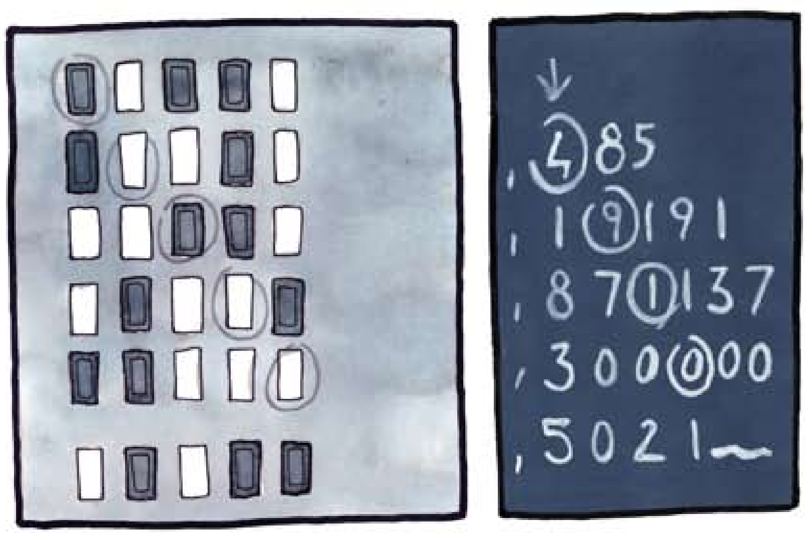


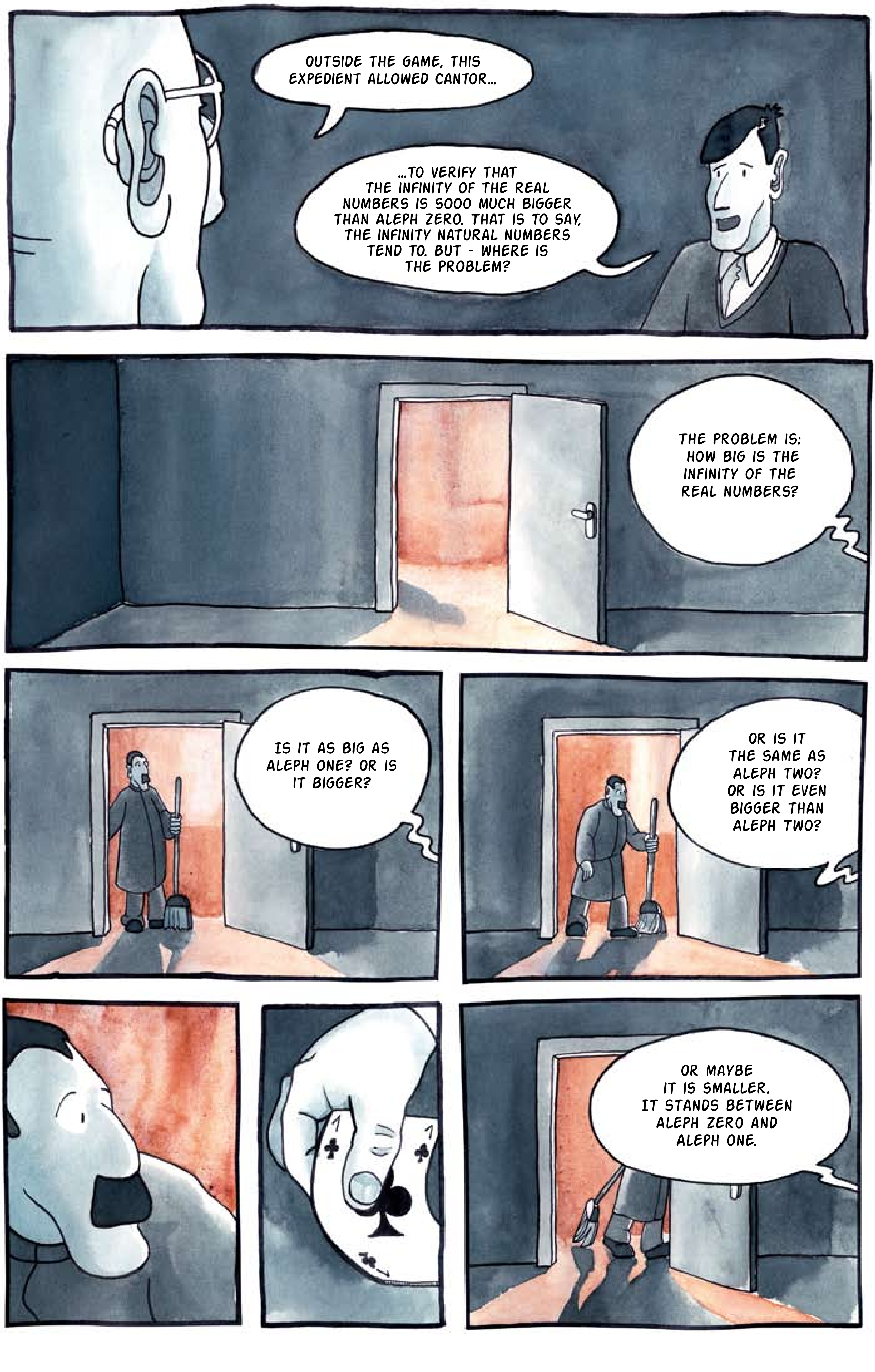

IT'S A MATTER OF CREATING A NEW SET STARTING FROM THE GIVEN ONES.
TRY TURNING THE CARDS ON THE DIAGONAL, ACCORDING TO CANTOR'S METHOD. JUST LIKE ON THE BLACKBOARD.


3
4
5

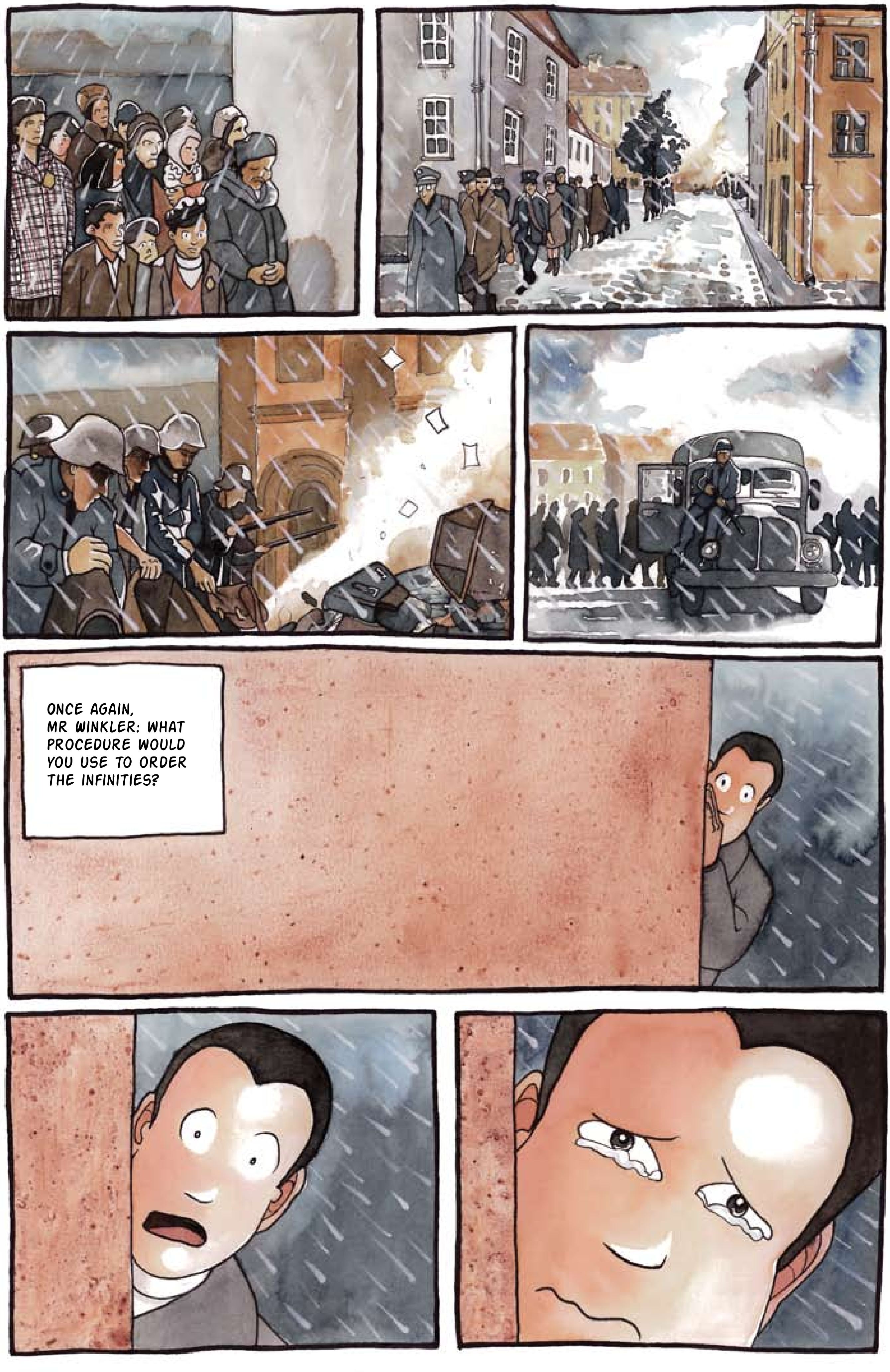


IT IS ALWAYS POSSIBLE TO CREATE A NEW SET, DIFFERENT FROM ALL THE GIVEN ONES.




OUTSIDE THE GAME, THIS EXPEDIENT ALLOWED CANTOR...
...TO VERIFY THAT THE INFINITY OF THE REAL NUMBERS IS SOOO MUCH BIGGER THAN ALEPH ZERO. THAT IS TO SAY, THE INFINITY NATURAL NUMBERS TEND TO. BUT - WHERE IS THE PROBLEM?
THE PROBLEM IS: HOW BIG IS THE INFINITY OF THE REAL NUMBERS?
IS IT AS BIG AS ALEPH ONE? OR IS IT BIGGER?
OR IS IT THE SAME AS ALEPH TWO? OR IS IT EVEN BIGGER THAN ALEPH TWO?
OR MAYBE IT IS SMALLER. IT STANDS BETWEEN ALEPH ZERO AND ALEPH ONE.


BEAR WITH ME A LITTLE LONGER. WHAT PROCEDURE WOULD YOU USE TO ORDER THIS SET?
GO AHEAD.
128 69 71 47 13 205 2
TO BEGIN WITH, I WOULD SINGLE OUT THE SMALLEST ELEMENT.
AND I WOULD TAKE IT OUT OF THE SET.
THEN I WOULD ONCE AGAIN SELECT THE SMALLEST OF THE REMAINING ELEMENTS.
AND, AGAIN, I WOULD TAKE IT OUT OF THE SET.
AND SO ON.
AND WHAT PROCEDURE WOULD YOU USE IF THE ELEMENTS WEREN'T NUMBERS BUT INFINITIES? AND IF THESE INFINITIES WERE INNUMERABLE?

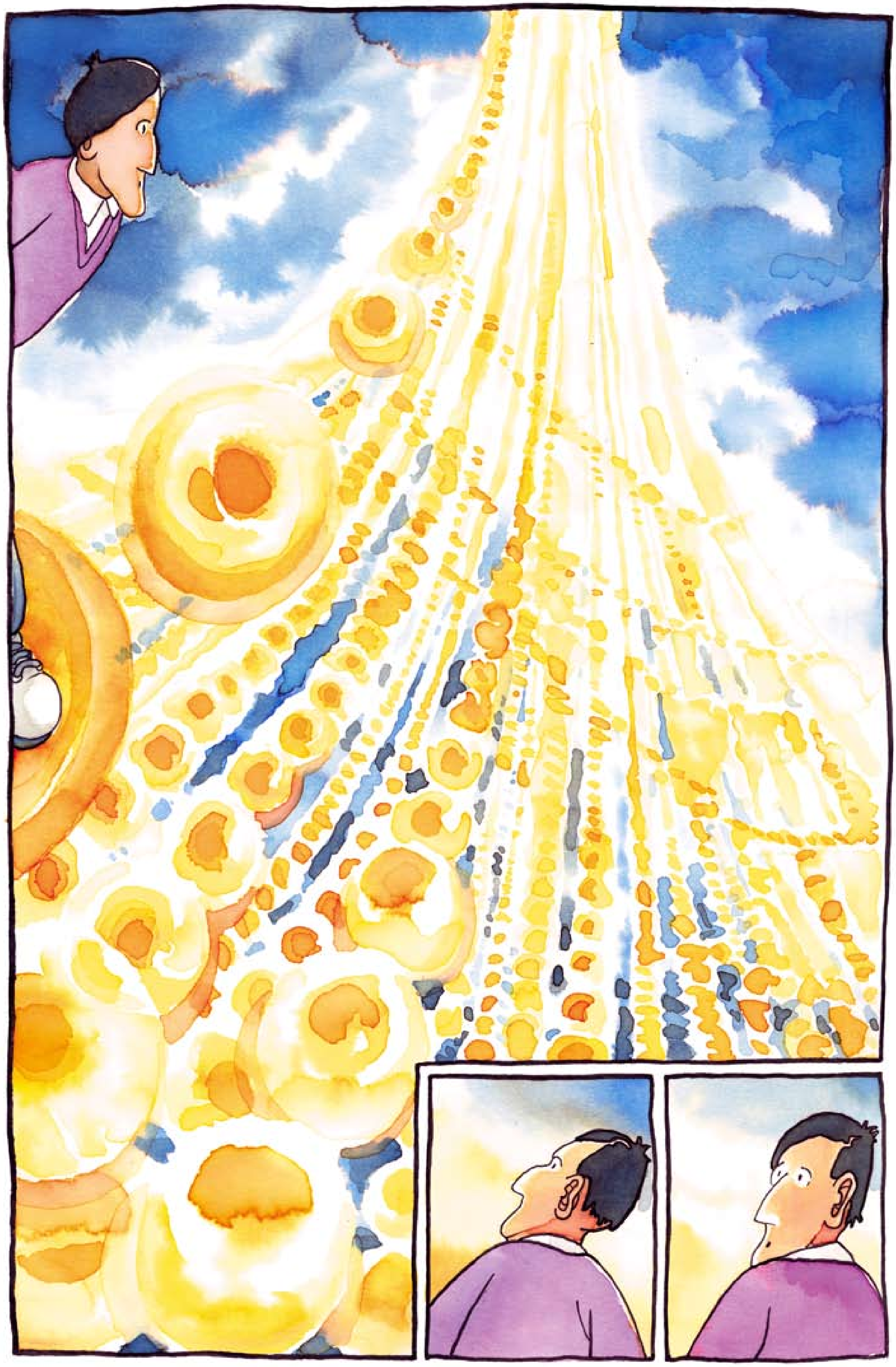

ONCE AGAIN, MR WINKLER: WHAT PROCEDURE WOULD YOU USE TO ORDER THE INFINITIES?

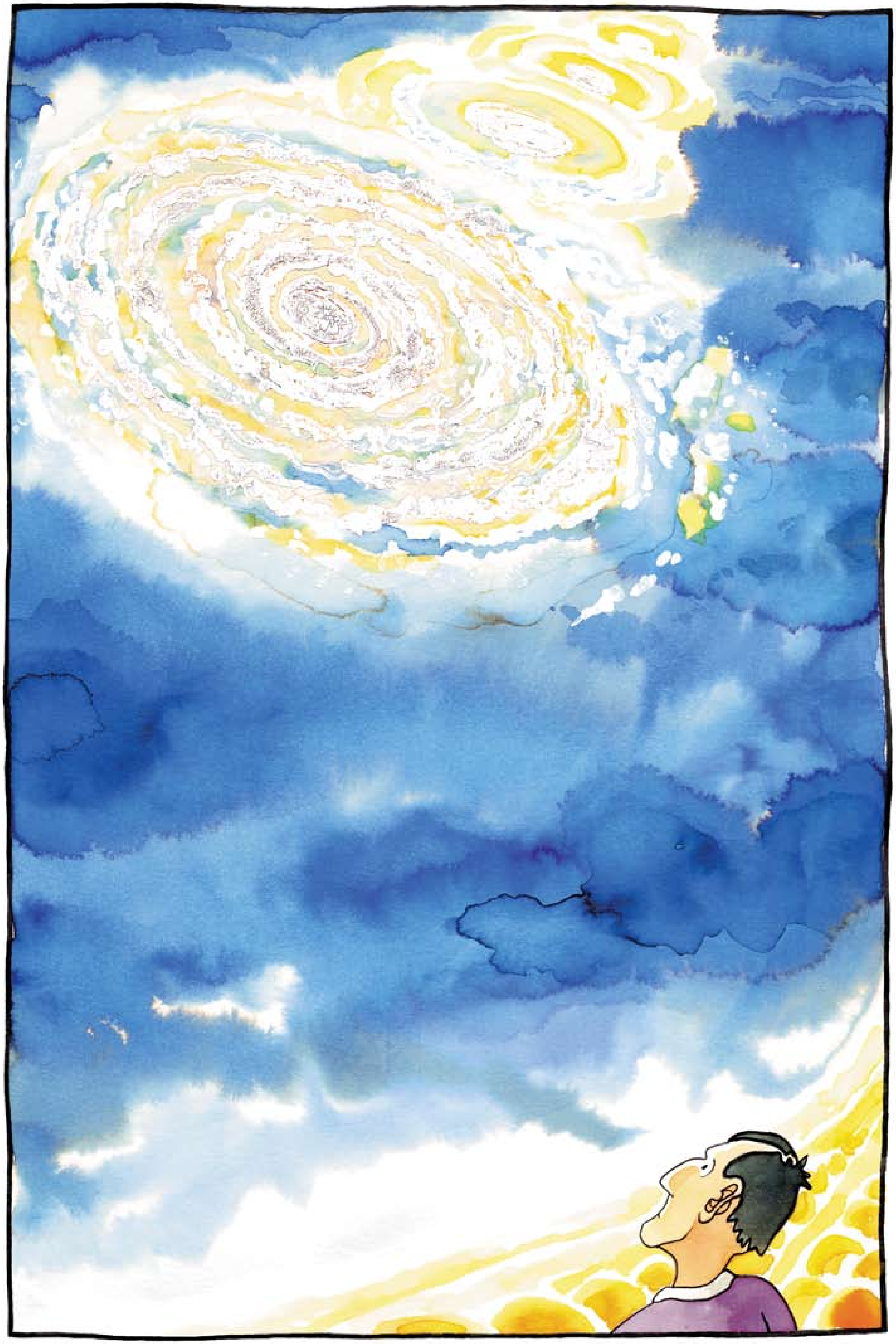

HERE IS WHERE THE CONTINUUM PROBLEM LIES: WE LACK A PROCEDURE THROUGH WHICH TO ORDER THE INFINITIES.


HAD IT BEEN THOUGHT OUT, THE CONTINUUM PROBLEM WOULD HAVE BEEN SOLVED, AND THE DENSE RANK OF GÖDELIAN PARADOXES WOULD HAVE LOST ITS MOST ILLUSTRIOUS PERSONAGE!


BUT IN THIS BIZARRE INVESTIGATION, IT SEEMS THAT THE CLOSER YOU GET TO THE HEART OF THE PROBLEM, THE MORE IT ESCAPES YOU.
BECAUSE THE PROCEDURE FOR ORDERING THE INFINITIES EXISTS. IT IS CALLED AXIOM OF CHOICE.
IT WAS POSTULATED BY OUR ILLUSTRIOUS COLLEAGUE DOCTOR ERNST ZERMELO.
BUT, ALAS, IT IS AN INFINITE PROCEDURE.
AN ORDER THAT ONLY ENDS AFTER INFINITE STEPS.
A SORT OF PROOF WHICH WOULD PROVE ONLY AFTER INFINITE ATTEMPTS.
AN INFINITE CYCLE.
A KIND OF PARADOX.


YES, ANOTHER PARADOX.
ANOTHER UNDECIDABILITY. ALWAYS THE SAME.
BUT THINGS EVOLVE AND FERMENT. GÖDEL STEPPED INTO THE CURRENT DEBATE WITH NEW ASTOUNDING HYPOTHESES.
IF YOUR PASSION FOR MATHEMATICS DOESN'T FADE, YOU'LL HAVE TIME TO GO DEEPER INTO THESE PURE AND DISTANT DISCIPLINES.
MY TIME HAS COME.
I WISH YOU AN INTENSE LIFE, MR WINKLER.
GOODBYE.
GOO - GOODBYE PROFESSOR FIZ.
TAP TAP






BROOOM
BROOOM
SCREEEC